A method for evaluating the performance of PSS with saturated input

H. Xin a,*, D. Gan a, T.S. Chung b, J. Qiu a

a Electrical Engineering College, Zhejiang University, Hangzhou, Zhejiang 310027, China
b Department of Electrical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Received 19 May 2006; received in revised form 30 July 2006; accepted 27 September 2006
Available online 13 November 2006

Abstract

In this paper, we report upon a method for estimating an ellipsoid, which resides entirely in the stability region of a linear system with saturation nonlinearities. To reduce the conservatism in the estimation, a procedure to transform the estimation problem into a simple convex optimization with linear matrix inequality (LMI) constraints is presented. As an application of this development, a method is introduced to estimate the stability region of a multi-machine power system with power system stabilizers (PSS) subject to saturated feedback. The idea is to check if the system state after a disturbance resides inside the estimated stability region. Thereby a sufficient condition is derived to conclude the effectiveness of saturated PSS controls. Numerical results of a test power system with three generators and five buses are described, indicating the reliability and simplicity of this approach.

© 2006 Elsevier B.V. All rights reserved.

Keywords: Saturated systems; PSS; LMI; Quadratic Lyapunov function

1. Introduction

Power system stabilizers (PSS) are commonly used to damp out angular oscillations in the range of 0.2–2.5 Hz associated with poorly damped swing modes [1–4]. When designing a PSS, the effect of saturation nonlinearities introduced by the PSS input limit should be studied as it unavoidably affects the performance of PSS.

Saturation nonlinearities are ubiquitous since the physical actuator or sensor is subject to saturation owing to its maximum and minimum limits [5–10]. Specially, most, if not all, power system controls are subject to saturation [1,11–12]. For example, the outputs of PSS and excitation control all have saturations, either intentionally designed or resulted from the limitations of equipments [2].

As is well-known, a linear system with a hyperbolic equilibrium is globally stable [13], but once saturation is considered, the system may not be globally stable and its stability region may also not be the global space but a neighborhood of the origin [5,13–15]. So if the actuator saturation exists, the performance of the closed-loop PSS control system designed without considering actuator saturation may seriously deteriorate and the PSS control system can only stabilize the perturbation whose initial state is in the stability region. Utility engineers did and should look at the issue, mainly relying on extensive simulation studies. Nevertheless, little attention has been paid to the impact of such actuator saturations on PSS control systems performance from analytical perspective, which is the main concern of this work.

The key is to estimate the stability region of PSS control system with actuator saturation. This task can be very complicated. On one side, due to the saturation nonlinearities, a simple linear system becomes a complex nonlinear system [15]; on the other side, a smooth (linear) system will become unsmooth [5,16], so theories applicable for smooth systems are invalid. In fact, most of the theories in dynamic systems are useful only for $C^r (r \geq 1)$ systems and the systems subject to windup saturation are only $C^0$ [16,17].

Since characterizing the stability region boundary of such systems seems impossible [5], the problem of estimating the stability region for such systems has been a focus of study in recent years [5,18–20]. Various results such as the circle and Popov criteria, see [18–21] for example, where the saturation is treated as a locally sector-bounded nonlinearity and the stability region is estimated by use of quadratic and lure type Lyapunov functions, have been developed. The circle

* Corresponding author.
E-mail addresses: huanhai_xin@163.com (H. Xin), dgan@zju.edu.cn (D. Gan), eetschun@polyu.edu.hk (T.S. Chung).
criterion is applicable to general memoryless sector-bounded nonlinearities, so we can expect the conservativeness in estimating stability region when it is applied to the saturation nonlinearity [5]. Based on Ref. [18], an ellipsoid inside the stability region, which is also in the linear space of the system, can be calculated by a quadratic Lyapunov Function. Also from this perspective, Ref. [5] derives less conservative conditions to ensure the ellipsoid reside inside the stability region by exploring the special property of the saturation function. This approach works for systems with very low dimensions only. The reason is that the number of such conditions would increase rapidly, making the approach computationally unattractive.

In this paper, a strict subset of stability region is estimated for a linear system with saturation nonlinearities by using a quadratic Lyapunov function based on the method described in Refs. [5,18]. To reduce the conservativeness in the estimation, we follow the procedure described in Ref. [5] to transform the problem of selecting an exact quadratic Lyapunov function into a simple convex optimization problem. The constraints in terms of LMI are presented by using the Schur complements of matrixes. Moreover, the procedure is very simple and can be handled efficiently by the toolbox in Matlab. As an application of this development, a methodology is introduced to estimate stability region of the PSS control system with saturation nonlinearity in power systems and to determine the effectiveness of the PSS. The idea is to check if the system state after a disturbance, say 5%, resides inside the estimated stability region. Thereby a sufficient condition is derived to conclude whether or not the PSS control under study is effective.

This paper is organized as follow. In Section 2, the model of a linear system with actuator saturation is presented. A methodology for estimating stability region is addressed in detail in Section 3. Applications of the methodology to power systems and its improved algorithms are described in Section 4. In Section 5, a numerical example for a power system with three generators and five buses is described, indicating the reliability and simplicity of this approach. Section 6 draws conclusions of this work.

2. The dynamics model with bounded input

Consider an open loop linear system [5,16]:

\[
\dot{x} = A'x + Bu
\]

\[
y = Cx
\]

where \(x \in \mathbb{R}^m\) is the state and \(u \in \mathbb{R}^m\) is the control. Here we consider a linear feedback. Suppose the control law is \(u = Gx\), where \(G \in \mathbb{R}^{m \times n}\), then the state equation of the closed loop dynamic system is:

\[
\dot{x} = (A' + BG)x = Ax
\]

The stability region of system (5) can be defined as [13]:

\[
\Omega = \{x \in \mathbb{R}^n | \phi_\Omega(x) \to 0\}
\]

where \(\phi_\Omega(x)\) denotes the trajectory of system (5) across point \(x\).

Since the function \(f(x)\) is continuous, \(\phi_\Omega(x)\) is also continuous [13]. Apparently, the stability region \(\Omega\) is the global state space if no actuator saturation exists, otherwise the stability region of system (5) may be some neighborhood of the origin. Obviously, if matrix \(A'\) in expression (1) has an eigenvalue with positive real part, the system (5) is not globally stable no matter what the control \(u\) is [5].

3. Region of attraction with bounded input

The expression (5) is the basic model for analyzing systems with input saturation. Unfortunately, the boundary of stability region cannot be characterized by analytic methods easily. So in this section, we give a method to estimate a strict subset \(\Omega_0\) of stability region.

First of all, we give the definition of set \(F\) and provide a result in Proposition 1.

\[
F = \{x \in \mathbb{R}^n | u \geq Gx \leq \bar{u}\}
\]

Assumption 1. \(A\) is a stable matrix.

Since \(A\) is the Jacobin matrix of the close loop system with the controller, the Assumption 1 is satisfied in engineering generally. Otherwise, the controller is invalid and it is not necessary to estimate the domain of attraction of this power system. Therefore, when we use this method in power system in Section 5, we did not determine whether Assumption 1 is satisfied or not.

Proposition 1. For system (5), the origin is an asymptotically stable equilibrium and there exists a neighborhood of the origin,
say $U$, which satisfies that $U \subset F$ and $U \subset \Omega$, i.e., $y < 0$, $\bar{u}_i > 0$ for all $i = 1, 2, \ldots, m$.

Remark 1. Since $A$ is the Jacobin matrix of the close loop system, the stability region does not exist if the proposition is not satisfied.

Choose a matrix $Q > 0$ (say $Q = I$, $I$ is the identity matrix). In this paper, expression $Q > 0$ denotes that $Q$ is a symmetrically positive matrix and $Q \geq 0$ denotes that $Q$ is a symmetrically non-negative matrix. Therefore, from Assumption 1, there exists a unique matrix $P > 0$ satisfying [13]:

\[ A^T P + PA = -Q \]  \hspace{1cm} (8)

Define set $\Omega_0$ as:

\[ \Omega_0 = \{ x \in R^n | V(x) \leq c \leq V_{cr} \} \]  \hspace{1cm} (9)

where $V(x) = x^T P x$, $V_{cr} = \min_{x \in \partial F} (V(x))$. Obviously, the differential function of $V(x)$ across the system (3) $V(x) = x^T (A^T P + PA) x = -x^T Q x$ is negative definite.

Lemma 1 and Theorem 1 below show that the set $\Omega_0$ is a strict set of stability region.

Lemma 1. Let $\varphi_i(x_0)$ denotes the trajectory of system (3) across $x_0$, if $\varphi_i(x_0) \in F$ for all $t \geq 0$ and for all $x_0 \in \tilde{\Omega}$, then the set $\tilde{\Omega}$ is a subset of the stability region $\Omega$ of system (5), i.e., $\tilde{\Omega} \subset \Omega$.

Proof. Since $\varphi_i(x_0) \in F$ is satisfied for all $x_0 \in \tilde{\Omega}$, the trajectory of system (3) is also that of system (5). By Proposition 1, the real parts of eigenvalues are negative, so system (3) is a globally stable system. Thus $\varphi_i(x_0) \in \phi_i(x_0) \in F$ and $\varphi_i(x_0) \to 0$ are satisfied, i.e., $x_0 \in \Omega$. Thus, $\tilde{\Omega} \subset \Omega$ and the lemma is proved.

Theorem 1. The set $\Omega_0$ defined in (9) is not only an invariant set but also a subset of set $F$ and $\Omega$, i.e., $\Omega_0 \subset \Omega$ and $\Omega_0 \subset F$.

Proof. According to the conclusion in Refs. [5,18], the set $\Omega_0$ is an invariant set and $\Omega_0 \subset F$. Obviously, the argument $\Omega_0 \subset \Omega$ can be easily verified by Lemma 1.

For system (5), Theorem 1 introduces a methodology to obtain a strict subset of stability region $\Omega_0$. Here we note that if the propositions are satisfied, the above theorem can be extended to deal with more general systems (10) as below:

\[ \dot{x} = Ax \]
\[ u \leq Gx \leq \bar{u} \]  \hspace{1cm} (10)

Therefore, to estimate an invariant set $\Omega_0 \subset F$, one can use the model (10) and follow the steps as follows (Algorithm 1):

1. Calculate the set $F$ as expression (7);
2. For system (5), check whether Assumption 1 is satisfied or not. If yes, continue; otherwise, exit the algorithm;
3. Calculate matrix $P$ from Eq. (8);
4. Solve the optimization problem:
\[ V_{cr} = \min_{x \in \partial F} (V(x)) \]  \hspace{1cm} (11)

Then the set $\Omega_0 \in \{ x \in R^n | V(x) < V_{cr} \}$ is an estimation of stability region, we seek.

From Algorithm 1, it can be seen that the key to obtaining $\Omega_0$ is to solve the quadratic programming problem (11). So we define set $E$ as:

\[ E = \bigcup_i (E_i \cup E_i') \]  \hspace{1cm} (12)

where $E_i = \{ x \in R^n | g_i x = \bar{u}_i \}$, $E_i = \{ x \in R^n | g_i x = u_i \}$, $g_i$ is the ith line of the matrix $G$ for $1 \leq i \leq m$.

Since the set $F$ is determined by some linear inequalities, the boundary $\partial F$ of set $F$ is a subset of $E$, i.e., $\partial F \subset E$. Thus if we let

\[ c_i = \min_{x \in E_i} (V(x)), \quad c^i = \min_{x \in E_i'} (V(x)), \quad i = 1, 2, \ldots, m \]  \hspace{1cm} (13)

\[ c = \min_{x \in E} (V(x)) = \min\{ c_1, \ldots, c_m, c^1, \ldots, c^m \} \]  \hspace{1cm} (14)

Then the inequality $c \leq V_{cr}$ is satisfied obviously. From Theorem 1, the set $\Omega_0$ is in the stability region of system (5) and the set $\Omega_0$ of a planar system is showed in Fig. 2.

Therefore, the optimization problem (11) is equivalent to a set of standard quadratic programming problems (13) and (14). Each of the quadratic programming problems takes the following form:

\[ \psi(d) = \min_{x} (V(x)) \]
\[ s.t. \quad g^T x = d \]  \hspace{1cm} (15)

For the above quadratic programming problem, we have the following result.

Lemma 2. The optimization result $\psi(d)$ of quadratic programming problem (15) is:

\[ \psi(d) = (g^T P^{-1} g)^{-1} d^2. \]  \hspace{1cm} (16)

Since the matrix $P$ is positive indefinite and $\psi(d)$ will not decrease when $|d|$ increases, the optimization problems (13) and (14) can be written as:

\[ c = \min\{ c(1), c(2), \ldots, c(m) \} \]
\[ s.t. \quad c(i) = (g_i P^{-1} g_i^T)^{-1} u_i^2 \]
\[ u_i = \min\{ -u_i, \bar{u}_i \} \]  \hspace{1cm} (17)

Now to estimate the stability region, we have (Algorithm 2):

1. The steps 1–3 of Algorithm 1;
2. Calculate the optimization problem (17) to obtain the set $\Omega_0 = \{ x \in R^n | V(x) \leq c \}$. 

Fig. 2. The set $\Omega_0$ for a planar system.
4. Application to power system PSS design

In the above section, we have introduced steps to estimate the stability region of linear systems with saturation. In this section, we will apply the method to estimate the stability region of a PSS control system with saturation nonlinearity, and then derive algorithms to determine the effectiveness of PSS control system. The idea is to check whether the initial state of power system after an expected disturbance, say 5%, resides inside the estimated stability region or not. Thereby a sufficient condition can be derived to conclude whether or not the PSS control under study is effective.

In a power system with PSS control, we assume that the measure of perturbation is $\beta$ (say 5%), namely the initial state $x_0$ of perturbation belongs to the set $X = \{x \in R^n | x^T x \leq \beta^2 \}$. Accordingly, to determine the effectiveness of PSS control in a power system, one can follow the following steps: (Algorithm 3):

1. Set system model as expression (10);
2. Apply Algorithm 1 or 2 to estimate the stability region $\Omega_0$;
3. Check whether the initial state is in set $\Omega_0$ or not, namely check the inequality $\beta \leq c^{-1} \lambda_{\text{min}}(P)$, and then draw the conclusion.

Apparently, among all the ellipsoids satisfying expression (9), we can expect that some ellipsoids are too conservative, so we would like to choose the best set with least conservatism by an optimization problem as what the Ref. [5] does. In this paper, the objective of the optimization is to acquire the biggest $\beta$ about perturbation which the PSS control can stabilize. Thus the optimization problem can be formulated as:

$$\beta^* = \max_{\beta > 0} \beta$$

s.t. (a) $X \subset \Omega_0 = \{x \in R^n | V(x) \leq c \}$
(b) $c \leq c(i) = (g_i P^{-1} g_i^T)^{-1} u_i^2$, $i = 1, 2, \ldots, m$
(c) $A^T P + P A < 0$

Where, the superscript “*” in $\beta^*$ means the optimized value of $\beta$. In the following text, this superscript will also be used to denote the corresponding optimized values in optimization problems.

**Remark 2.** Since the variable $c$ is determined by the constraint (b), the result of expression (18) must satisfy that there exists some index, say $k$, such that $c = c(k)$ for all $1 \leq k \leq m$. Namely, the derived stability region $\Omega_0$ is tangent with the $k$th constraint $g_k x = u_k$ as shown in the Fig. 2, which also can be illustrated by the following simulation studies. Here the $k$th constraint $g_k x = u_k$ is termed as the active constraint.

**Remark 3.** In problem (18), some constraints are strict inequality, which may result the admissible region not being a closed domain, so the optimization problem cannot obtain the optimal solution and the model may be defective. To overcome this defect, in our paper we seek sub-optimal solutions by replacing the inequality $P > 0$ with $P \geq \epsilon I$, where $\epsilon$ is a small (tiny) number which can be tolerated and it is taken by experience (in this paper, $\epsilon$ is set to $10^{-6}$).

To handle the optimization problem (18) efficiently, the Schur complement tool is used to transform the nonlinear and convex constraints (a), (b) and (c) into LMIs. Suppose $Q > 0$, then,

$$\begin{bmatrix} R & S \\ S^T & Q \end{bmatrix} \geq 0$$

if and only if

$$R - S Q^{-1} S^T \geq 0.$$

Since $P$ is a symmetric positive definite matrix, by the Schur complements, constraint (a) is equivalent to:

$$x \in \Omega_0 \iff \{x | x^T (\beta^{-2} I) x \leq 1 \} \subset \{x | x^T(c^{-1} P)x < 1 \}$$

$$\implies \beta^{-2} I - c^{-1} P \geq 0 \iff \begin{bmatrix} \beta^{-2} I & I \\ I & c P^{-1} \end{bmatrix} \geq 0$$

And constraint (b) is equivalent to:

$$\begin{bmatrix} u_i^2 - g_i (c P^{-1}) g_i^T & 0 \\ 0 & 1 \end{bmatrix} \geq 0$$

$$\iff \begin{bmatrix} 1 \\ c P^{-1} \end{bmatrix} \begin{bmatrix} u_i^2 & g_i \\ g_i^T & c^{-1} P \end{bmatrix} \begin{bmatrix} 1 \\ c P^{-1} \end{bmatrix}^T \geq 0$$

(20)

And also by Schur complement, constraint (c) is equivalent to:

$$c P^{-1} A^T + c A P^{-1} < 0$$

(21)

Let

$$R = c P^{-1}, \ Z = [z_1^T, z_2^T, \ldots, z_m^T]^T = K R,$$

$$\gamma = \beta^{-2}, \ z_i = g_i R, \ i = 1, 2, \ldots, m$$

following expressions (19)–(22), the problem (18) can be transformed to a standard convex optimization with LMIs constraints and can be rewritten as:

$$\gamma^* = \inf_{R > 0} \gamma$$

(23)

s.t. (a) \[ \begin{bmatrix} \gamma I & I \\ I & R \end{bmatrix} \geq 0 \]

(b) \[ \begin{bmatrix} u_i^2 & z_i \\ z_i^T & R \end{bmatrix} \geq 0, \ i = 1, 2, \ldots, m \]

(c) $RA^T + AR < 0$

There are plenty of algorithms to handle linear program with LMI constraints. In our work, we use the function of
LMI-toolbox in Matlab to solve it. Therefore, to analyze the effectiveness of PSS with saturated input, one can follow the following steps (Algorithm 4):

(1) Solve the power flow and linearize the system dynamic equations at the equilibrium to obtain the linear system model as (10);
(2) Solve the optimization problem (23) to derive $\gamma^*$, $R^*$, and $Z^*$ by Mincx function in the toolbox of Matlab, and then calculate $\beta^*$, $P^*$ by the inverse transformation of expression (22).
(3) Compare the expected perturbation $\beta_0$ and the value $\beta^*$ obtained form step (2). The PSS control is effective when $\beta_0 \leq \beta^*$ is satisfied.

Remark 4. If constraint (a) in optimization problem (23) is changed to

$$\gamma P_0 \leq 0$$

then optimization problem (23) has the following meaning: the estimated stability region encloses the largest ellipse with fixed-shape reference set which is $X_0 = \{x|x^T P_0 x \leq \beta^2\}[5,23]$. Thus this method can be easily extended to solve the problem that the anticipated set of initial state values may be similar to a fixed-type ellipse since the state variables are changing at different speed and direction during the perturbation of power system. In this paper, we only consider, for illustration purpose, the situation where the shape reference set of initial states is a special ellipse, that is, a ball.

5. Simulation results

5.1. The model of PSS with saturated input

In this section, we will use a test system with detailed models to illustrate the algorithms derived in previous sections.

Suppose that the power system under study consists of $N_b$ buses and $n_g$ generators, we take impedance model for loads, the simple AVR model as shown in Fig. 3 for the excitation system of generators [1], the model as shown in Fig. 4 where the input is $\Delta \omega_1$ and the output is the excitation signal for the PSS control system.

Consider the following saturated constraints:

(1) The output $\Delta E_f$ of generator’s excitation;
(2) The output $y_2$ of the PSS control system.

Then the closed loop system after linearization can be expressed as:

$$\dot{y} = \tilde{A} y, \ u \leq \tilde{G} y \leq \bar{u}$$

where,

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & \bar{A} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}; \quad \tilde{u} = \begin{bmatrix} \bar{u}^T \\ \bar{u}^T \\ \bar{u}^T \end{bmatrix}$$

and

$$\bar{A} = \begin{bmatrix} 0 & -I & 0 \\ M^T K_1 & M^T D & M^T K_2 \\ T_{d0}^T K_1 & 0 & T_{d0}^T K_2 \end{bmatrix}; \quad \bar{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$$

The definitions of the variables in (24)–(27) are partly labeled in the Figs. 3 and 4, which also can be found in Ref. [1]; the variables $M$, $T_{d0}$ and $T_A$ are the diagonal matrixes of the corresponding parameters of generators; the variables $K_1$, $T_1$, $T_2$ and $T_w$ are also the diagonal matrixes of which the elements are the corresponding variables of PSS system.

In system (26), since there exists a zero eigenvalue and specially two zero eigenvalues with the uniform damping coefficients, which violates Assumption 1. A reference machine, say the $N_g$th generator, can be chosen to overcome this problem, which can be easily done for the system with uniform damping coefficients via the following transformation:

$$A = S\tilde{A} \cdot \text{pinv}(S), \ G = \tilde{G} \cdot \text{pinv}(S), \ x = Sy$$

Fig. 3. Excitation system with AVR.

Fig. 4. The diagram of the PSSs transfer function.
Here, 
\[ S = \text{diag}\{s_1, s_1, s_2\}, s_1 = |I_{(ng-1)\times(ng-1)}|e], s_2 = I_{4ng\times4ng}, e = \begin{bmatrix} -[1, 1, \ldots, 1]^T \end{bmatrix} \]
and the symbol \text{pinv}(\cdot) denotes the generalized inverse function [22].

Thus, from (24)–(28), the degenerately equivalent model of (26) can be rewritten as:
\[ \dot{x} = Ax, \quad u - \bar{u} \leq Gx \leq \bar{u} \quad (29) \]

The above model satisfies the main assumption thus can be used to analyze the effectiveness of PSS control system.

5.2. Examples

The test power system is composed of three generators and five buses, whose parameters are shown in the Fig. 5. The parameters of each PSS are shown in Appendix A.

For the test system, by Algorithm 4, we can obtain the variable \( \gamma = 148.11 \), namely, the most severe perturbation that the PSS control system can stabilize is equal to \( \beta^* = \max(\beta) = 8.22\% \).

We can also obtain the stability region of the system with these saturated constraints and the corresponding figures of stability region on some planar coordinates are the areas enclosed by red dashed as shown in the Fig. 6.

Thus, from the above analysis in Section 4, we can derive a sufficient condition that though the outside perturbation has resulted excursion of initial value from origin, as long as the excursion lies in the globe whose radius is 8.22\%, the system is stable since the PSS can stabilize the perturbation, which also can be proved by simulation as shown in the Fig. 7.

If we apply Algorithm 2 but not Algorithm 4, the derived variable \( \beta^* \) is only 0.347\% which is very conservative and is much smaller than 5\%, commonly used in the engineering. Moreover, the mapped area of stability region by Algorithm 2 is also smaller than that obtained by Algorithm 4. Accordingly, Algorithm 4 not only enlarges the measure of stability region, but also minimizes the differences among the axes of the estimated ellipsoid so that the biggest hyperball can be contained in the ellipsoid. This is the same as the conclusion in Section 4.

In Fig. 6E, which is the projection of stability on the coordinate \( \Delta E_{f3} - \Delta E_{f1} \), we note that the excitation constraints of generators 1 and 3 are tangent with the stability region at point a and point c, respectively. Hence, the active constraints are the excitation saturation constraints of generators 1 and 3. Namely, the inequality constraints (b) become equality constraints when the index i is equal 1 or 3 in the expression (23).

When the saturated value \( u_3 \) of generator 3 is changed from 5.2 to 7.2 and the other parameters are fixed, the corresponding shapes of stability region will be changed rapidly. Fig. 6 plots these projections, which are the ellipsoids depicted by blue real line. From Fig. 6A–C and E, we can also note that the areas enclosed by real lines will increase along with the increase of
while the areas enclosed by real line in Fig. 6D and F decrease along with the increase of $u_3$. This is because the objective of Algorithm 4 is to make the estimated stability region enclose a hyperball with the biggest radius.

When a constraint is relaxed, say $k$th, a constraints that are not the active constraints will turn to the active constraints, which induces the optimal value of $P^*$ to be changed and the estimated ellipsoid to be reshaped, thus the axes of this ellipsoid will alter with not the same rate.

In the same manner, the change of other parameters will also influence the estimated stability region, and so the variable $\beta^*$ that PSS can stabilize. Fig. 8 shows the relationships between the variable $\beta^*$ and $u_i$ of the corresponding constraints, where the data of Fig. 8 are shown in Appendix B.

In Fig. 8, we can conclude that: for the system with actuator saturation, when Assumption 1 is not satisfied, i.e., $u_i \leq 0$, the estimated stability region will not exist, so the value of $\beta^*$ is zero; when $u_i > 0$, the value of $\beta^*$ increases linearly along with the increase of $u_i$; when $u_i$ arrives at some point such the point ‘A’ in the first graph of Fig. 8, the value of $\beta^*$ will not change.

The above phenomenon can be explained from the following analysis. When the value of $u_i$ is small, the active constraint is the $i$th hyper-plane $g_i x = u_i$, so the optimized value $P^*$ is a constant matrix. Moreover, the relationship between $\beta^*$ and the parameter $u_i$ is $\beta^* = u_i \lambda_{\min}(P^*)$, therefore the optimized value $\beta^*$ is a linear function with respect to variable $u_i$, where, $\lambda_{\min}(P^*)$ denotes the minimum eigenvalue of matrix $P^*$. But when the value of $u_i$ increases to its turn point, say point A in the first graph of Fig. 8, the other constraints, say the $j$th constraint, will become active, when the value $\beta^*$ is a nonlinear function of parameter $u_i$. After the value of $u_i$ is increased continuously to some extent, the $i$th constraint will not be an active constraint and the value $\beta^*$ is determined by the others constraints and is independent of the $i$th constraint $g_i x = u_i$, so the value of $\beta^*$ will be a constant along with the increase of parameter $u_i$.

6. Conclusions

In this paper, we give a sufficient condition under which a PSS control can stabilize a given perturbation and then present a simple method to analyze the effectiveness of PSS control with saturated input in power system. Since power systems are high-dimension dynamic systems, the optimization model described in this study may have dimension disaster problem. Further work should be towards reducing the conservativeness of stability estimation and avoiding the dimension disaster problem. Extending
the idea from linear systems to nonlinear systems is also another work worth further efforts.

Acknowledgements

This research is jointly supported by the Natural Science Foundation (Project 50595411) of China and Department of Education of China under New Century Outstanding Investigator program. This research is also partially supported by Hong Kong Polytechnic Research Grant #A-PG54.

Appendix A. The data of the excitation and PSS

<table>
<thead>
<tr>
<th>$K_A$</th>
<th>$T_A(s)$</th>
<th>$K_s$</th>
<th>$T_s(s)$</th>
<th>$T_1(s)$</th>
<th>$T_2(s)$</th>
<th>$\Delta E_f$</th>
<th>$\Delta E_f$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>6</td>
<td>0.05</td>
<td>1.0</td>
<td>5.0</td>
<td>0.35</td>
<td>0.03</td>
<td>0.35</td>
<td>6.2</td>
<td>7.0</td>
<td>1.5</td>
</tr>
<tr>
<td>G2</td>
<td>10</td>
<td>0.05</td>
<td>1.0</td>
<td>8.3</td>
<td>0.35</td>
<td>0.03</td>
<td>0.35</td>
<td>5.2</td>
<td>6.0</td>
<td>1.5</td>
</tr>
<tr>
<td>G3</td>
<td>17.5</td>
<td>0.05</td>
<td>1.0</td>
<td>8.0</td>
<td>0.45</td>
<td>0.05</td>
<td>0.45</td>
<td>5.2</td>
<td>6.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Appendix B. The data of Fig. 8

<table>
<thead>
<tr>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
<th>$u_5$</th>
<th>$u_6$</th>
<th>$K_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>variable</td>
<td>4.2</td>
<td>6.2</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>B</td>
<td>variable</td>
<td>3.5</td>
<td>7.5</td>
<td>2.0</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>5.2</td>
<td>variable</td>
<td>6.2</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>5.4</td>
<td>6.0</td>
<td>variable</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>4.2</td>
<td>5.2</td>
<td>variable</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>F</td>
<td>5.0</td>
<td>4.5</td>
<td>10.5</td>
<td>1.0</td>
<td>1.5</td>
<td>variable</td>
</tr>
</tbody>
</table>

References


Huanhai Xin is a Ph.D. candidate in the College of Electrical Engineering, Zhejiang University, China. His research interests include power system stability analysis and control.

Deqiang Gan has been with the faculty of Zhejiang University, since 2002. He visited the University of Hong Kong, in 2004. He was a Senior Analyst in ISO New England, Inc. from 1998 to 2002. He held research positions in Birkbeck University, University of Central Florida, and Cornell University from 1994 to 1998. He received a Ph.D. in Electrical Engineering from Xian Jiaotong University, China, in 1994. He is currently on editorial board of European Transactions on Electric Power. His research interests include the applications of nonlinear system and game theory to power systems.

Tak-shing Chung received his B.Sc., M.Sc., and Ph.D. degrees from The University of Hong Kong, Imperial College of Science and Technology, UK and University of Strathclyde, UK, respectively. He has been with the Department of Electrical Engineering, Hong Kong Polytechnic, since 1977. He is presently a Professor and Leader of Power Group in Department of Electrical Engineering. His research interests include power systems control, optimization, stability and expert system applications.

Jiaju Qiu is a professor of Electrical Engineering in Zhejiang University. He received a Ph.D. in 1988 from the same university where he teaches and conducts research in the general area of power engineering. Professor Qiu has over 10 years of industry experiences. He visited the University of Strathclyde and Cornell University in 1993, 1994, respectively.