A Self-Organizing Strategy for Power Flow Control of Photovoltaic Generators in a Distribution Network

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Abstract—The focus of this paper is to develop a distributed control algorithm that will regulate the power output of multiple photovoltaic generators (PVs) in a distribution network. To this end, the cooperative control methodology from network control theory is used to make a group of PV generators converge and operate at certain (or the same) ratio of available power, which is determined by the status of the distribution network and the PV generators. The proposed control only requires asynchronous information intermittently from neighboring PV generators, making a communication network among the PV units both simple and necessary. The minimum requirement on communication topologies is also prescribed for the proposed control. It is shown that the proposed analysis and design methodology has the advantages that the corresponding communication networks are local, their topology can be time varying, and their bandwidth may be limited. These features enable PV generators to have both self-organizing and adaptive coordination properties even under adverse conditions. The proposed method is simulated using the IEEE standard 34-bus distribution network.

Index Terms—Distribution network, network control, power flow control, PV generators, self-organization.

I. INTRODUCTION

In recent years, there has been an increasing number of distributed generators (DGs) integrated into the modern distribution network [1]–[3]. Among these DGs, PVs are becoming an increasingly attractive source of renewable energy in certain areas, especially as they become more commercially viable to manufacture and install [4], [5]. However, the introduction of a large amount of PV generation could have a negative impact on the power system at large if the relevant stability and control issues are not properly considered.

In a distribution network that contains a high level of penetration of PV units but has an insufficient number of storage devices, a sudden disturbance (such as the change in sunlight due to a passing cloud/storm) could trigger a rapid disconnection of these PV generators or the reduction of their operating capacity, resulting in a significant loss of active power support. The loss of generation can lead to an array of problems, such as voltage variation [4], [6], transient stability issues, and even voltage collapse [7]–[9], making it desirable to develop a practical and robust scheme of controlling the total output of the PVs, especially in the case that the distribution network is weak.

The optimal power flow (OPF) strategy is the standard approach to dispatch and control traditional generators in the transmission network [10]–[12]. The strategy has also been applied to distribution networks that have a few DGs [13], [14]. These results yield centralized controls by collecting the system-wide information and sending the command globally. For a power system whose transmission and distribution networks have numerous and geographically dispersed DGs, such centralized controls are too expensive to be implemented and, even if they could be done, the resulting system is not robust or efficient.

On the other hand, a decentralized control mode, such as the maximum photovoltaic power tracking (MPPT), constant voltage and frequency (VF) with droop mode, or the feeder power flow control mode [1], [2], [15], [16], is useful in the distribution network if there are only a few DGs present.

As the level of PV penetration increases, intermittent changes of these PV outputs become too many to enumerate and consider, and hence, it becomes intractable for either the decentralized or centralized mode to adequately manage a distribution network, such as maintaining the voltage profile. Instead, the practically feasible solution is a distributed control configuration [17] that uses local communication networks and hence combines the positive features of both centralized and decentralized controls while limiting their disadvantages. Specifically, a PV generator should incorporate whatever information is available from its neighboring generators or critical nodes into its control law. This distributed control configuration is in line with the concept that a future smart grid would include communication among network elements [3], [4]. This resulting control problem becomes the so-called network control problem which has systematically been studied in recent years and also successfully applied to such fields as cooperative robotic control [18], [19].

The distinctive feature of an appropriately designed network control is that it admits local and changing communication networks, is robust with respect to intermittency and latency of its feedbacks, and also tolerates connection and disconnection of network components. When a distribution network contains many groups of DGs whose output may vary in wide ranges,
the aggregated power output of all the DGs in each group can be dispatched and controlled by coordinating the power output level of each DG internally within the given group. For instance, a simple solution to the power-output coordination problem is to prescribe certain utilization profile for all of the PVs in a group, and the so-called fair utilization profile is that the same ratio of power output versus available power is imposed on all individual PV generators. In general, the desired control algorithm should be able to manage a large number of DGs but require only local information from neighboring units. By doing so, groups of DGs can be formed autonomously based on the presence of local communication networks, the power in a distributed network can be easily dispatched and controlled, and possibly large swings in power outputs of distributed generators can be tolerated.

In this paper, a self-organizing power control method is proposed by applying cooperative control theory of networked systems to power distribution networks. The proposed control algorithm can effectively control a large number of PVs which exchange information among their neighboring units, respectively. The control can make the PVs in one group converge to any given utilization profile while providing the desired power being dispatched from the group. It is distinctive that the proposed control can tolerate the changes in the distribution network, its local communication network can be intermittent and have time varying topologies, and its communication bandwidth can be of minimum. Typical communication setups are discussed, and the necessary and sufficient condition for stability and convergence is also presented.

II. PROBLEM FORMULATION

A. Dynamical Model

Consider a distribution power system network with \( n \) three-phase inverter-based photovoltaic generators that use the decoupled d-q control method via phase locked loops (PLL). Under the d-q frame, the terminal voltage of the \( i \)th PV generator satisfies \( U_{di} = U_i \) and \( U_{qi} = 0 \), where \( U_i \) denotes the magnitude of the terminal voltage [15]. Thus, output power of the \( i \)th PV generator can be expressed as

\[
P_i = U_i I_{di}, \quad Q_i = -U_i I_{qi}
\]

where \( I_{di} \) and \( I_{qi} \) are the output currents in the d-axis and q-axis, respectively; and \( P_i \) and \( Q_i \) denote the active and reactive power.

If an integration control design is pursued for \( P_i^{ref} \) and \( Q_i^{ref} \), then the dynamics of the distribution system are described by the following differential-algebraic equations:

\[
P_i^{ref} = u_{1i}
\]

\[
Q_i^{ref} = u_{2i}
\]

\[
\dot{\chi} = g_1(P_1, \ldots, P_n, Q_1, \ldots, Q_n, \chi, \mathbf{X})
\]

\[
P_i = U_i I_{di}, \quad Q_i = -U_i I_{qi}
\]

\[
0 = g_2(P_1, \ldots, P_n, Q_1, \ldots, Q_n, \chi, \mathbf{X})
\]

where (2) and (3) denote the d-loop and q-loop dynamics through which the active and reactive power outputs can be controlled; variables \( u_{1i} \) and \( u_{2i} \) are the control inputs to be designed; \( \chi \) is a vector of an appropriate dimension, and it denotes all the internal network state variables including dynamics of the PV generators, loads, and synchronized generators; \( \mathbf{X} \) is a vector of algebraic variables in the distribution network such as the voltages of buses; and (6) is the power flow equation of the distribution network.

Controls \( u_{1i} \) and \( u_{2i} \) \((i = 1, 2, \ldots, n)\) are to be designed to control active and reactive power outputs of PV generators so that the power flow of the network is reasonable. Toward this goal, the changes of the PVs’ outputs can be considered to be much slower than the dynamics of PVs’ internal variables, and hence, it can be assumed that the dynamics of (4) have been made asymptotically stable by their controls. In other words, fast dynamics of \( \chi \) can be considered negligible compared to the slowly changing power flow variables. Accordingly, the simplified dynamical models for the distribution network are denoted by

\[
P_i^{ref} = u_{1i}
\]

\[
Q_i^{ref} = u_{2i}
\]

\[
P_i = U_i I_{di}, \quad Q_i = -U_i I_{qi}
\]

\[
0 = g(P_1, \ldots, P_n, Q_1, \ldots, Q_n, \chi, \mathbf{X})
\]

where \( g(\cdot) = [g_1^{T}(\cdot), g_2^{T}(\cdot)]^{T} \), and all the other variables are the same as those in (2)–(6). In what follows, a distributed cooperative control will be designed for system (7)–(10).

B. Problems to be Solved

In a distribution network with many PV generators, its operating condition needs to be adjusted due to many varying factors, such as load and sunlight fluctuations. As was pointed out in the introduction, it would be impossible to determine and maintain a feasible operating condition if all of the PV generators are independently run under the decentralized control configuration shown in Fig. 1(a). The centralized control mode shown in Fig. 1(b) is neither practical nor reliable since it requires global collection and exchange of information. To handle numerous PV generators in the network, we propose to implement the distributed control configuration as shown in Fig. 1(c).

The basic idea of the proposed distributed control is that, by incorporating local communication networks that share information among neighboring units, numerous PV generators autonomously form a number of generation groups and that each group is represented by a virtual generation unit which aggregates all the power generated within. The outputs of these virtual generators can be dispatched and controlled by a high-level control (which is equivalent to a transmission and distribution control center). While the latter is similar to the centralized control configuration, the proposed control configuration requires much less communication since the numerous distributed generators do not communicate with the higher-level control.

Once the power output of each group is dispatched, the proposed cooperative control coordinates all the outputs of PV generators in the same group so that, even though the output ca-
capacity of individual PVs may have large swings, a given profile is achieved for their utilization and the sum of their outputs converges to the dispatched value. This feature makes it possible for all the active PVs to self-organize themselves and be controlled. The local information sharing within one group of PVs may be intermittent, asynchronous, and of varying topology. In other words, the proposed control is robust with respect to possible variations and limitations of communication networks. The utilization profile for PVs in the group can be determined according to such considerations as economic and regulatory policies. In what follows, the proposed distributed cooperative control is designed for the case of the fair utilization profile, that is, that all of the PVs in a designated group are to be run at the same active and reactive power output ratios. Mathematically, this control problem can be stated as follows.

**Problem 1:** Design controls $u_{i1}$ and $u_{i2}$ ($i = 1, 2, \ldots, n$) for system (7)-(10) such that, at the equilibrium operating point, the utilization profile of PVs is described by the following ratios:

\[
\frac{P_i}{P_{i\text{max}}} = \frac{P_{i2}}{P_{i2\text{max}}} = \ldots = \frac{P_n}{P_{n\text{max}}} = \alpha^0_P
\]

and

\[
\frac{Q_i}{Q_{i\text{max}}} = \frac{Q_{i2}}{Q_{i2\text{max}}} = \ldots = \frac{Q_n}{Q_{n\text{max}}} = \alpha^0_Q
\]

where $\alpha^0 = [\alpha^0_P, \alpha^0_Q]^T$ is the given output ratio for the active and reactive power for all the PV generators, respectively; $P_{i\text{max}}$ and $Q_{i\text{max}}$ are the instantaneous maximum capacity of $P_i$ and $Q_i$, respectively; and all the $P_{i\text{max}}$ are assumed to have the same sign, and so do all the $Q_{i\text{max}}$.

The ratios in (11) and (12) are the utilization percentages of all the PV units and they are commanded to be the same at the steady state. Accordingly, the control objective of (11) and (12) is referred to as the fair utilization profile. In general, a distribution network can dictate any specific utilization profile. For instance, utilization ratio $\alpha_i$ may be desired to converge to $\alpha_i^* \neq 0$ for different $i$. In this case, the designer can introduce the gain of $k_i = \alpha_i^*/\alpha_i^*$ so that the transformed utilization ratio of $k_iQ_i$ needs to converge to the common ratio $\alpha_i^*$. In other words, any specific utilization profile can be converted into the fair utilization profile in (11) and (12) for which the proposed cooperative control is designed. It should be noted that a balanced distribution network is considered in problem 1. If the distribution network is unbalanced, the energy management should be different because the output power contains a periodic part resulting from the negative current and voltage. For this case, the power in (11) and (12) should be replaced by the positive output power.

By introducing the utilization profile (11) and (12), each of the PVs can be controlled by comparing its operation to that of any or some of its neighboring units, and there is no need for every PV unit to communicate with every other unit or the high-level control. Accordingly, the proposed distributed control for each PV generator is of the general form

\[
u_i = u_i(s_{i0}y_0, s_{i1}y_1, s_{i2}y_2, \ldots, s_{in}y_n), \quad i = 1, 2, \ldots, n
\]

where $y_0$ denotes the output of the high-level control; $y_i$ represents the output of the $i$th PV generator (for $i = 1, 2, \ldots, n$); $S = (s_{ij})$ is the matrix defined below to describe the instantaneous communication topology:

\[
S = \begin{bmatrix}
s_{10}(t) & s_{11}(t) & s_{12}(t) & \ldots & s_{1n}(t) \\
s_{20}(t) & s_{21}(t) & s_{22}(t) & \ldots & s_{2n}(t) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
s_{n0}(t) & s_{n1}(t) & s_{n2}(t) & \ldots & s_{nn}(t)
\end{bmatrix} \in \mathbb{R}^{n \times (n+1)}.
\]

In (14), $s_{ij}(t) \equiv 1$ for all $i$; $s_{ij} = 1$ if the output of the $j$th PV generator is known to the $i$th PV generator at time $t$, and $s_{ij}(t) = 0$ if otherwise; $s_{j0}(t) = 1$ if the $j$th PV generator receives information from the high-level control at time $t$, and $s_{j0}(t) = 0$ if otherwise. As shown in Fig. 1(c), the proposed control utilizes certain neighboring information but does not assume any global information. In the extreme case that the high-level control collects information from and sends command to all the PVs, matrix $S$ in (14) has 1 as its entries in the first column, and the corresponding control becomes the centralized control configuration in Fig. 1(b). The other extreme case is that none of the PVs exchange information with each other or the high-level control, then matrix $S$ has entries $s_{ij} = 0$ for any $i \neq j$, and the proposed control reduces to a decentralized control.

The specific expression of the control (13) will be synthesized in Section IV-A. In both the control (13) and matrix $S$ of (14), the identity of $s_{ij} \equiv 1$ means that the control at every PV generator can and should utilize its own output as one of the
feedback. Whether or not the high-level command or information from other PV generators is available at the \( t \)th generator is determined by the current status of its communication network represented by the \( t \)th row of communication matrix \( S \). Intuitively, it would be sufficient for all the PV generation units to be controlled properly if each of the PVs can receive some information from some of its neighboring units and if the command of the total dispatched power is sent to some of the PVs in a group. This brings out the problem of determining the minimum information sharing that must exist among the group of PV generators and between the group and the high-level control. That is, the requirement on communication network needs to be specified to ensure the proposed distributed control would work, and it is stated as the following problem.

**Problem 2:** Determine how the local communication network among the PVs (the \( S \) matrix) should be designed in order to ensure efficiency and reliability of the network while minimizing economic costs.

It should be emphasized that the proposed distributed control admits time varying, intermittent, and asynchronous communication networks. In fact, given any limited bandwidth of one communication node, it would be better for the convergence of networked control that the node sends its information intermittently to different nodes in its vicinity. This means that, in (14), communication matrix \( S \) is time-varying in general or, more precisely, piecewise constant. Specifically, consider

\[
t_{\infty,0} := \{t_0, t_1, t_2, \ldots\}, \quad S_{\infty,0} := \{S(t_0), S(t_1), \ldots\}
\]

which means that \( S \) changes at \( t_k \) \((t_0 = 0)\), or simply, \( S(t) = S(t_k) \in S_{\infty,0} \) for \( t \in [t_k, t_{k+1}) \). The time-varying nature of matrix \( S \) also implies that the proposed distributed control has inherent robustness against interruptions, data loss, and other phenomena of a typical communication network.

Problem 2 defined above arises naturally from the fact that both the communication network topology and the availability of PVs are time varying. In addressing the problem, we aim to find a simple condition on feasible and convergent sequences of \( \{S(t_0), S(t_1), \ldots\} \). The basic rule for designing appropriate local communication topology will further be discussed in Section IV. Through such a design of local communication network, the candidate sequences of \( \{S(t_0), S(t_1), \ldots\} \) can be chosen to ensure the convergence of the distribution network under the proposed control. Due to the time-varying nature of local communication networks and their resulting cooperative controls, the resulting closed loop system around the equilibrium point is piecewise-constant, and its stability has to be analyzed in terms of convergence of matrix sequences. In other words, the proposed control can be designed by using the state-of-the-art approach from cooperative control of networked systems rather than traditional methods such as eigen-analysis and pole placement.

Upon the solutions to Problems 1 and 2 being found, all the PVs are organized into a number of groups and, within each group, the PV generators are controlled to satisfy the given utilization profile. Accordingly, each group of the PVs can be viewed as a virtual generator of a larger capacity and with an aggregated output. The aggregated outputs of these virtual generators would be dispatched and in turn the operating ratios in group’s utilization profiles are determined, which results in much less long-distance communication and also a much simpler design for a high-level control [see that in Fig. 1(c)] to address ancillary service issues, given the large numbers of PVs in the distribution network.

Thus, depending upon the needs of the particular distribution network of our concern, the high-level control can be designed according to one of several objectives. Among the possibilities, the high-level control can be designed to be either a balanced generator in the islanding operation, or a virtual power plant for load management [20], or a smart agent for maintaining frequency or a desired voltage profile [1], [6], and so on. Relevant to this paper is the study in [21] where the power at the feeder is kept constant such that all the rest of the load demands are picked up by DGs. In this case, the feeder becomes a truly dispatchable load from the utility side, allowing demand-side management arrangements. In this paper, the idea of making the load constant to the utility grid is extended; in particular, the high-level control is designed so that the reactive power is controlled to support the voltage at some critical bus and to keep the active power consumed by loads in an area or at a feeder to be constant. This leads to the third problem to be solved in the paper.

**Problem 3:** Based on the solutions to Problems 1 and 2, design a high-level control [as shown in Fig. 1(c)] for the system of (7)–(10) such that, for each group of PVs, the voltage of a critical bus is of a specific value and the active power flow across certain transmission line is of a fixed amount. Namely, the following set points are achieved at the equilibrium:

\[
\begin{align*}
P_{\text{tran}}(\cdot) &= P^{\text{ref}} \\
V_c(\cdot) &= V^{\text{ref}}
\end{align*}
\]

where \( P^{\text{ref}} \) and \( V^{\text{ref}} \) are the given reference values; \( V_c(\cdot) \) and \( P_{\text{tran}}(\cdot) \) represent the voltage of the selected bus and the active power over the chosen transmission line, respectively. Note that, in (16) and (17), variables \( V_c(\cdot) \) and \( P_{\text{tran}}(\cdot) \) are functions of the outputs of PVs and, for simplicity, the functional dependence is denoted by (\cdot\).

Several observations are worth making here regarding the aforementioned design problems.

1) The design problems 1–3 are defined explicitly in terms of dynamics of PV generators, and they can be extended by incorporating dynamics of other types of DGs. That is, for a distribution network with other types of DGs such as wind generators, the same design problems can be formulated and solved for energy management. As argued in [21], a hierarchical control architecture should be used to control the distribution network with many DGs. In this regard, Problem 3 is the high-level control, Problems 1 and 2 are the middle-level control within each of the groups of DGs, and also there are low-level individual controls [which are already embedded into (7)–(10)] at each of the DGs to control the individual output. To apply such, the DGs can easily be classified into groups according to their
location and proximity, for instance, at a feeder, within a communication zone, solar/wind farms, etc. How to best partition DGs into groups is an important problem, but it is out of the scope of this paper. In our simulation study, all the PVs connected to the same feeder are considered to belong to one group.

2) In Problems 2 and 3, a DG group is controlled to act as a virtual FACTs element supporting the voltage of a critical bus. In general, it is difficult to determine the critical bus for a large-scale power system. The common practice is by simulations or by experience. For a radial distribution network, existing simulation studies [2], [6] have shown that the bus at the end of a feeder is typically the one subject to voltage violation when power from DGs changes intermittently. Thus, in our simulations, the end bus of a feeder is chosen to be the critical bus.

3) Problem 3 aims to coordinate any given DG group with the overall power grid. The desired active and reactive power output (i.e., reference values $P_{\text{ref}}$ and $V_{\text{ref}}$) of the PV group can be determined by either offline or online optimization approach or by one of the market-based approaches [6], [13], [22]. Once $P_{\text{ref}}$ and $V_{\text{ref}}$ are obtained, the extended power flow equations of (6) and of (16) and (17) could be used to determine the group’s aggregated power output (which in turn yields the actual output ratios of the active and reactive power, $\alpha_P^0$ and $\alpha_Q^0$). If the high-level control could collect enough information (including the parameters of the power system network with virtual DG generators), the desired output ratios of every PV group can be calculated directly. Otherwise, the high-level control for system coordination needs to be designed as prescribed in Section IV-B.

III. RULE OF COMMUNICATION TOPOLOGY DESIGN

As outlined in the previous section, the topology of the local communication network among the PVs in one group can be intermittent (time varying) and asynchronous, in which case the communication matrix $S$ describing the topology becomes piecewise constant. To ensure that a given utilization profile is achieved among one group of PVs, there need to be local information sharing among the DGs. Heuristically, the more communication channels there are, the more information propagates within the group, and the faster the convergence to the desired utilization profile. However, this quickly becomes an uneconomical solution to the problem. It follows from cooperative control theory [23] that the minimum requirement on the communication topologies is the so-called sequential completeness condition. Mathematically, this requirement is that the sequence of communication matrices $S_{\infty,0} = \{S(t_0), S(t_1), \ldots\}$ be sequentially complete in the sense that, over an infinite sequence of finite consecutive intervals, the composite graph over each of the intervals (or the binary product of all the matrices of $S$ over the interval) has at least one globally reachable node (in the sense that all other nodes can be reached from the globally reachable node by following the directed branches of the graph) [23]. Thus, for the purpose of distributed control design, the following rule is given for designing local communication networks and scheduling local communications.

Rule: Communication matrix $S$ is piecewise constant, and the corresponding sequence $S_{\infty,0} = \{S(t_0), S(t_1), \ldots\}$ is sequentially complete.

The above sequential completeness condition is a very precise method to schedule local communication, and it is also the necessary and sufficient condition for any properly-designed cooperative system to converge [18], [23]. A more restrictive (sufficient but not necessary) condition is that the composite graph is strongly connected (which implies that, by following the directed branches, every node can be reached from any other node). To illustrate its application, consider communication matrix $S(t_k)$ and construct the corresponding graph by linking the nodes according to nonzero entries in $S$. One can easily determine whether the resulting graph has at least one globally reachable node or not. For instance, consider the graphs in Fig. 2. Fig. 2(a) has node 0 as the unique globally reachable node; and none of the nodes in Fig. 2(b) is globally reachable because there are two isolated groups of nodes.

Fig. 3 plots the graphs corresponding to those communication topologies. It follows from Fig. 3 that the information can propagate from node 0 to nodes 1, 2, and 3. Therefore, all of the communication matrices are complete by themselves, and so are their sequences.

While not necessarily required, this special case can be used to design and implement a redundant local communication network which satisfies the so-called rule of “N-n”.
when \( n \) communication channels cannot work properly in some amount of time, the communication matrix corresponding to the remaining communication channels should be kept to be complete. It should be also noted that the convergence rate of the closed loop system depends upon connectivity of the communication network, so it is important to design a reasonably connected local communication network within certain physical and economic constraints.

### IV. Cooperative Control Strategy

#### A. Control Strategy for Fair Utilization

In this subsection, the high-level control is assumed to have sufficient information, such as the exact parameters and the (aggregated) states of the distribution network, so that the desired output ratios \( \alpha_P^0 \) and \( \alpha_Q^0 \) can be computed directly using the expanded power flow equations. This assumption will be removed in the subsequent subsection.

It follows from (7)–(9) that

\[
\hat{P}_i = \hat{U}_i \hat{P}_i + U_i P_i + U_i \hat{u}_i \\
\hat{Q}_i = -\hat{U}_i \hat{Q}_i - U_i \hat{Q}_i + U_i \hat{u}_i.
\]

The cooperative control laws for the \( i \)th PV generator are

\[
\hat{u}_i = \frac{K_{d_i} P_i}{U_i} \left( D_{i0} \alpha_P^0 - \frac{P_i}{P_i^{\text{max}}} + \sum_{j=1}^{n} \frac{D_{ij} P_j}{P_j^{\text{max}}} \right) - \frac{U_i P_i}{U_i^{\text{max}}} \\
\hat{u}_i = \frac{K_{q_i} Q_i}{U_i} \left( D_{i0} \alpha_Q^0 - \frac{Q_i}{Q_i^{\text{max}}} + \sum_{j=1}^{n} \frac{D_{ij} Q_j}{Q_j^{\text{max}}} \right) + \frac{U_i Q_i}{U_i^{\text{max}}} \tag{20}
\]

where \( K_{d_i} \) and \( K_{q_i} \) are the gains

\[
D_{ij} = \frac{s_{ij}}{\sum_{j=0}^{n} s_{ij}}, \quad D_{ij}^0 = \frac{s_{ij}^0}{\sum_{j=0}^{n} s_{ij}^0}, \quad i = 1, 2, \ldots, n \tag{22}
\]

and \( s_{ij} \) is the generic entry of matrix \( S \) defined in (14) for \( u_{\ell_1} \).

Similarly, \( S' \) can be defined for \( u_{\ell_2} \) and \( s_{ij}' \) is its generic entry.

Without loss of any generality, let us choose for simplicity that

\[
K_{d_1} = K_{q_1} = \cdots K_{d_n} = K_{q_n} = K_0. \tag{23}
\]

Then, it follows from (18)–(21) and (23) that the closed loop system becomes

\[
\frac{\hat{P}_i}{P_i^{\text{max}}} = K_0 \left[ -\frac{P_i}{P_i^{\text{max}}} + D_{i0} \alpha_P^0 + \sum_{j=1}^{n} \frac{D_{ij} P_j}{P_j^{\text{max}}} \right] \tag{24}
\]

\[
\frac{\hat{Q}_i}{Q_i^{\text{max}}} = K_0 \left[ -\frac{Q_i}{Q_i^{\text{max}}} + D_{i0} \alpha_Q^0 + \sum_{j=1}^{n} \frac{D_{ij} Q_j}{Q_j^{\text{max}}} \right]. \tag{25}
\]

The following theorem shows that, if the PV generators are controlled by (20) and (21), their active and reactive power utilization profiles converge to the desired values defined in (11) and (12), respectively.

**Theorem 1:** Consider control laws (20) and (21) and suppose that the communication rule is satisfied among the PVs. Then, the output ratios of their active and reactive power converge uniformly and asymptotically to the common values of \( \alpha_P^0 \) and \( \alpha_Q^0 \), respectively.

**Proof:** System (24) can be rewritten as

\[
\hat{z}_i = K_0 \left[ -z_i + D_{i0} \alpha_P^0 + \sum_{j=1}^{n} D_{ij} z_j \right]. \tag{26}
\]

where \( z_i = P_i/P_i^{\text{max}} \). On the other hand, constant \( \alpha_P^0 \) is the trajectory of the following virtual system:

\[
z_0 = 0, \quad z_0(0) = \alpha_P^0. \tag{27}
\]

Therefore, stability of system (26) is identical to that of the following extended system consisting of (26) and (27):

\[
\begin{cases}
\dot{z}_0 = 0, \quad z_0(0) = \alpha_P^0 \\
\dot{z}_i = K_0 \left[ -z_i + \sum_{j=0}^{n} D_{ij} z_j \right].
\end{cases} \tag{28}
\]

It follows from [23, theorem 5.4] that, if communication matrix \( D \) defined by (22) is sequentially complete, system (28) uniformly asymptotically converges to \( c1 \), where \( 1 = [1,1,\ldots,1]^T \) and \( c \in R \) is the constant determined by the initial state and topology changes. It follows from \( z_0 \equiv \alpha_P^0 \) that \( c = \alpha_P^0 \). Thus, \( z = [z_1,\ldots,z_n]^T \) uniformly asymptotically converges to \( \alpha_P^0 1 \). Similarly, reactive power ratios also converge to the common value of \( \alpha_Q^0 \). □

Theorem 1 provides the solution to both Problems 1 and 2. The salient features of the proposed distributed control have been described in Sections II and III, and they will be illustrated in Section V.
Mathematically, by choosing gains $K_p$ and $K_v$ to be small enough, then the desired utilization issued by the high-level controls changes much slower than the outputs of the PVs. It is known [23] that the system in the form of (28) is exponentially convergent to $\alpha^0_Q, 1$ and it is robust with respect to any disturbance in terms of $[z_i - z_j]$ for every pair of $\{i, j\}$. Accordingly, [23, theorem 5.4] can be used to conclude that, due to the separation of time scales, system (24) and (25) remains to be asymptotically convergent (to the common desired utilization ratios) and so is the closed loop system under the entire set of distributed control (33). This result provides the solution to Problems 1, 2, and 3. 

The following two remarks are worth noting about the distributed high-level controls. 1) PVs can also be used to damp possible oscillations existed in a power system network or to improve the long-term stability by designing the additional control inputs $\Delta P_{\text{additional}}$ and $\Delta V_{\text{additional}}$ (shown in Figs. 4 and 5) in a way similar to the PSS and AVR controls for synchronous generators. This subject will be studied in future research, but in this paper, these additional inputs are set to zero. 2) The high-level controls can also be implemented discretely, that is, by using the simple rule that, if

$$
\begin{align*}
\alpha^0_Q(t) &= \alpha^0_Q(t_k) + \Delta z_Q^0 \\
\times \text{sign}(V_{\text{ref}}(t_k) - V_c(t_k)) \\
\alpha^0_P(t) &= \alpha^0_P(t_k)
\end{align*}
$$

where $\Delta z_Q^0$ and $\Delta z_P$ are the step sizes in adjusting the utilization ratios; $\text{sign}(\cdot)$ is the standard sign function; $t_k. (i = 1, 2, 3, \ldots)$ are the time instants of updating.

C. Further Discussions on Distributed Control

In addition to the distinct features already described in Sections II and III, the following observations are also worth noting about the proposed distributed controls.

1) In the distributed controls, $P_{\text{trans}}$ is assumed to be nonzero. In the event that some of the PV generators are out of service (due to weather or otherwise), the apparent singularity of $P_{\text{trans}} = 0$ can be avoided by using the simple rule that, if $P_{\text{trans}} < \varepsilon_0$, the $j$th column of communication matrix $S$ is set to zero. That is, a PV generator in any group is ignored unless it has meaningful power generation capacity.

2) It has been shown that $s_k(t) = 1$ is not required for all the PVs. This means that the interaction between the distributed high-level controls and distributed cooperative controls can also be intermittent and time varying. In particular, when the dispatched power over certain transmission lines is changed or critical bus voltages are adjusted, only their reference values in the high-level controllers need to be modified.

These two features imply that the proposed two-level distributed controls are very flexible and effective, which enable the groups of PVs to be truly self-organizing according to their
capacity and local communication networks. Should more paths be added into the local communication networks, the faster the system converges, which will be illustrated in the simulation study.

V. SIMULATION STUDY

In this section, the standard IEEE 34-bus distribution network is used to illustrate effectiveness of the proposed distributed two-level controls. The network has its main voltage at 24.9 kV. Its topology is shown in Fig. 6. Its DGs include 16 PV generators and one gas turbine generator. Simulations are done using Digsilent, and balanced distribution networks are considered.

The parameters of the system can be found at the link http://ewh.ieee.org/soc/pes/dsacon/testfeeders.pdf. Basic operational conditions, dynamical models, and other settings are:

**PV parameters**: The initial maximums of active and reactive power are set to 1 MW and 1 MVAR, respectively.

**Loads**: 1.45 MW + j0.85 MVAR. Their model is composite (a dynamical part and a constant impedance part of 50% each).

**Gas turbine generator**: Its active power is 2 MW and the terminal voltage is set to 1.0 p.u.; the default model in Digsilent is used with IEEE Type 1 control for AVR, PSS_ovestab1 model for the PSS, and the IEEE-G1 model for the governor.

**PV generators**: 3.2 MW + j0.16 MVAR (\(\cos \phi \approx 1.0\)).

**External grid**: An infinite bus (1.0 p.u.).

The parameters in the aforementioned dynamical models are included in the Appendix.

**High-level controls**: The voltage is controlled for the critical bus to which PV 1 is connected (recall that, as discussed in Section IV, the last bus in a feeder is typically chosen for a radial network with intermittent DGs); active power control is to keep a constant amount of power move downstream of the feeder (line 1 and its positive direction are shown in Fig. 6).

**Matrix of communication topology**: The local communication network is represented by matrix \( S \) in (34) where \( G_i \) denotes the \( i \)-th PV generator (\( i = 1, 2, \ldots, 16 \)) and \( G_n \) is the high-level controller (which coordinates the aggregated power and hence is considered to be a virtual PV generator in the lead). Unless mentioned otherwise, matrix \( S \) of (34) is the one used in the simulation:

\[
S = \begin{bmatrix}
G_0 & G_1 & G_2 & G_3 & G_6 & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 1 & 1 & 1 & 0 \\
\end{bmatrix}
\]

As specified by (34), the 1st and the 2nd PV generators receive the information from the high-level control; generators G1, G2, G3, and G6 are closely coupled, and the rest of generators follow G2 up to G6 by receiving information from them.

It is easy to verify that the aforementioned communication topology satisfies the communication rule presented in Section III. In fact, the communication topology is intentionally chosen to be redundant so that, if some of the entries in matrix \( S \) were to switch from 1 to 0 intermittently, the communication rule should still be observed unless many of the communication channels stop working at the same time. This shows that the local communication network can be designed to be robust.

In this distribution network, the distributed generators (including the gas turbine generator) provide power to not only the local loads but also the external main grid. In the simulation, total penetration levels of DGs and PVs are to be increased up to about 360% and 220%, respectively. The corresponding voltages at the PV terminal buses are shown in Table I with different PV penetration levels (provided that all the PVs are run according to the fair utilization profile and their power factors are also kept to be same). From this table, one can see that, if the penetration level of PVs is raised further, the bus voltage...
TABLE I

<table>
<thead>
<tr>
<th>PV</th>
<th>0%</th>
<th>50%</th>
<th>100%</th>
<th>220%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.912</td>
<td>0.947</td>
<td>0.978</td>
<td>1.046</td>
</tr>
<tr>
<td>2</td>
<td>0.912</td>
<td>0.947</td>
<td>0.978</td>
<td>1.046</td>
</tr>
<tr>
<td>3</td>
<td>0.950</td>
<td>0.986</td>
<td>1.023</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.924</td>
<td>0.955</td>
<td>0.983</td>
<td>1.043</td>
</tr>
<tr>
<td>5</td>
<td>0.939</td>
<td>0.999</td>
<td>1.039</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.912</td>
<td>0.978</td>
<td>1.046</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.912</td>
<td>0.978</td>
<td>1.046</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.912</td>
<td>0.978</td>
<td>1.044</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.950</td>
<td>0.966</td>
<td>1.021</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.951</td>
<td>0.986</td>
<td>1.021</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.966</td>
<td>0.985</td>
<td>1.002</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.916</td>
<td>0.949</td>
<td>1.042</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.913</td>
<td>0.978</td>
<td>1.044</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.912</td>
<td>0.978</td>
<td>1.044</td>
<td></td>
</tr>
</tbody>
</table>

shall go out of the feasible range (0.9 p.u.−1.1 p.u.). Thus, unless being controlled appropriately, operations of this system will have voltage problems under certain disturbances.

The subsequent simulation study is to verify that, under disturbances and interruptions, the proposed distributed two-layer control can make the system operate well and all the PV generators’ outputs converge according to the desired utilization profile. The following cases are simulated:

1) short-circuit faults;
2) changes in sunlight;
3) load changes and intermittent communication interruptions.

A. Dynamic Responses to Short Circuit Faults

The fault location is labeled by point (a) or the associated red dot in Fig. 6. The fault at point (a) occurs at 0.0 s and is cleared after 0.12 s. The resulting dynamical responses of the network are shown in Fig. 7.

Fig. 7 shows that the system is asymptotically stable under this fault and the corresponding time scales are in seconds. In comparison, dynamics of internal state variables such as those of synchronized generators are much faster than the changes in PV generators’ outputs. This validates the assumption (made in Section II) that, for the power control problem, those internal dynamics can be ignored.

It can also be observed that, at the time that the fault occurs, the active power generated by the PV generators decreases, and so does the active power across line 1. Accordingly, the high-level control commands PV 1 and PV 2 to increase their power output ratios and, as the result of the proposed cooperative control, all the rest of PV generators follows. At the end, all the PV generators asymptotically reach the fair utilization profile. This transient process is plotted in Fig. 7(3). Reactive power is controlled in a similar way, as shown in Fig. 7(4).

B. Dynamic Responses to Rapid Changes in Sunlight

Next, sudden changes of available power due to weather conditions are considered. In the simulation, the maximum active power capacity of PV 1, PV 3, and PV 5 are to vary between 10% and 100%, as shown in Fig. 8(1).

Fig. 8 shows the dynamical responses of the system. In particular, the active/reactive power output ratios are shown in Fig. 8(2) and (3), respectively; and the power transmitted over line 1 and the bus voltages are plotted in Fig. 8(4) and (5), respectively.

When the sunlight reaching a PV generator starts to decrease, the active power output ratio initially increases since the maximum available power of the corresponding PV generator is decreased. This process is apparent in Fig. 8(2) and (3) in which the active power output follows the trend of changes in sunlight except for the transient responses due to the aforementioned reason and to the controls.

As sunlight changes, the distributed cooperative control makes PV output ratios converge to new operating points determined by the distributed high-level controls. The high-level controls do not know the current generation capacity of all the PV generators since they communicate only with PV generators 1 and 2. Nonetheless, during the whole period of system operation, the active power sent over line 1 and the bus voltages are maintained around their designated values, which are shown in Fig. 8(4) and (5). This confirms the features claimed in the previous discussions.

Under the proposed power flow control method, the feeder can be considered from the viewpoint of bus (b) to be a virtual emulated load of constant active power, and the inner voltage of this feeder can be controlled by reactive power control of the PV generators. Fig. 9 shows the changes of real and reactive power output ratios. Therefore, the effects due to the fluctuations of the
PV real power generation can be reduced or minimized by implementing local information sharing and cooperative controls. It is believed that the proposed method can be applied to solve voltage regulation problems.

C. Load Changes and Communication Interruptions

The previous simulations show that PV generators can autonomously adjust their outputs to converge to a prescribed utilization profile (i.e., the fair utilization profile). In what follows, a case is studied to show that the proposed control also has strong robustness against communication interruptions and load variations.

Consider the following severe case that local communication network is interrupted intermittently and the loads vary simultaneously.

1) The simulated communication interruptions are chosen as follows: the communication channels of $0 \rightarrow 2$, $1 \rightarrow 3$, and $2 \rightarrow 10$ are intermittent, and the time instants of their changes are random (whose changes are shown in Fig. 10(1), in which “1” means that the corresponding channel is working and “0” otherwise).

2) All loads ramp to 200% of the base levels within 1 s, and 10% loads of the base loads are cleared at 5 s and 10 s, respectively.

Dynamical responses of the gas-turbine generator and the PV generators are shown in Fig. 11, in which the dashed and solid trajectories represent the results without or with communication interruptions. The output ratios of representative PV generators...
(PVs 1–3) are plotted in Fig. 12. As expected, we see by comparing solid and dashed trajectories that convergence is faster without communication interruptions.

VI. CONCLUSION

A distributed two-level control scheme is proposed for power control of groups of PV generators in a distribution network. The proposed control guarantees that the distribution network can autonomously adjust itself to a feasible operating point (after disturbances occur) and that all the PV generators in one group converge to a prescribed utilization profile. The proposed controls require only intermittent information sharing among neighboring generators, and topologies of local communication networks can be time varying. As long as the communication networks meet the minimum information exchange requirement (of the matrix sequence being sequentially complete), the proposed controls ensure convergence. Simulations of the IEEE benchmark distribution network are used to validate the features and effectiveness of the proposed method. The proposed design methodology is also applicable to distribution networks with different types of DGs including solar-, wind-, and ocean-energy power generators.

APPENDIX

The parameters for the gas turbine generator are: $S_B = 5$ MVA, $H = 1.5$ s, $Damp = 0.02$ p.u., $x_1 = 0.1$ p.u., $x_d = 2.002$ p.u., $x_q = 2.002$ p.u., $T''_{d0} = 5.51$ s, $T''_{f0} = 0.8$ s, $T''_{d0} = T''_{f0} = 0.1$ s, $x''_d = x''_q = 0.171$ p.u.

The AVR model in the gas turbine generator is the IEEE type 1 [24]. Its transfer function is shown in Fig. 13 and the parameters are: $T_f = 0.02$ s, $K_f = 200$ p.u., $T_a = 0.03$ s, $K_e = 1$ p.u., $T_e = 0.2$ s, $K_f = 0.05$ p.u., $T_f = 1.5$ s, $V_{\text{min}} = -10$ p.u., $V_{\text{max}} = 10$ p.u.

The PSS in the gas turbine generator is the PSS_ostab2 model provided by the Digsilent. The transfer function is shown in Fig. 14, in which the parameters are: $K = 200$ p.u., $T_1 = T_2 = 0.125$ s, $T_3 = T_4 = 0.025$ s, $K_{ang} = 8$ p.u., $T_{ang} = 0.5$ p.u., $T_w = 0.02$ s, $L_{\text{min}} = -0.2$ p.u., $V_{\text{ngmax}} = 0.29$ p.u., $V_{\text{ngmax}} = 0.05$ p.u., $V_{\text{max}} = 0.2$ p.u., $V_{\text{min}} = -0.2$ p.u.

The governor in the gas turbine generator is the IEEE-G2 model, the transfer function is shown in Fig. 15, and the parameters are: $K = 5$ s, $T_1 = 0.5$ s, $T_2 = T_3 = 0.1$ s, $T_3 = 0.95$ s, $P_{\text{min}} = 0$ p.u., $P_{\text{max}} = 1.0$ p.u.

The parameters for the PV generators are: $K_P = 0.1$ p.u., $K_V = 0.5$ p.u., $K_d = K_q = 8.0$ p.u., $i = 1, \ldots, 16$.

The dynamical model for the loads is shown in Fig. 16, and the parameters are: $T_1 = 0.1$ s, $k_{pf} = k_{qf} = 1.0$ p.u., $k_{pt} = k_{qt} = 1.0$ p.u., $T_{pf} = T_{qf} = 0.05$ s, $T_{pu} = T_{qu} = 0.05$ s.

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