Process Control

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Zhejiang University 2016
• Most popular control method in industry is proportional-integral-derivative or PID controller, around 95% of industrial problems.

• Apparent simplicity of understanding and generally satisfactory performance.

• Based on action on the process according to the error between the set point and the measured output.

• Numerous variants of PID exist. Improvements possible by better tuning.
Process representation in *open loop*

Open loop can work only if the process model is perfect and in the absence of disturbances, noise.
Process representation in *closed loop*

Objective: maintain the output $y$ as close as possible to the desired set point $y_r$ for any disturbance.
- The output is measured using a given measurement device; the value indicated by the sensor is $y_m$.

- This value is compared to the set point $y_r$, giving the difference [set point – measurement] to produce the error $e = y_r - y_m$.

- The value of this error $e$ is provided to the main corrector to modify the control variable $u$. 
A controller represents a control strategy, i.e. a set of rules providing a value of the control action when the output deviates from the set point.

A controller can thus be constituted by an equation or an algorithm.

In this first stage, only simple conventional controllers are considered.
Proportional (P) Controller

- Operating output of the proportional controller is proportional to the error

\[ u_a(t) = K_c e(t) + u_{ab} \]  \hspace{1cm} (1)

where \( K_c \) is the proportional gain of the controller. 
\( u_{ab} \) is the bias signal of the actuator (\( u_{ab} \) = operating signal when \( e(t) = 0 \)).
- The gain \( K_c \) can be positive or negative.
- The gain \( K_c \) has dimensions.
- The higher the gain, the more sensitive the controller.
- A controller can saturate when its output \( u_a(t) \) reaches a maximum \( u_{a,\text{max}} \) or minimum \( u_{a,\text{min}} \) value.
- Controller transfer function

\[ G_c(s) = K_c \]  \hspace{1cm} (2)

simply equal to the controller gain.
- *Philosophy: the integral action takes into account the present.*
Proportional-Integral (PI) Controller

• The operating output of the PI controller is proportional to the weighted sum of the magnitude and of the integral of the error

\[ u_a(t) = K_c \left( e(t) + \frac{1}{\tau_I} \int_0^t e(x) \, dx \right) + u_{ab} \]  

(3)

• \( \tau_I \) in the integral time constant.

• The integral action tends to modify the controller output \( u_a(t) \) as long as an error exists in the process output.

• Transfer function of the PI controller is equal to

\[ G_c(s) = K_c \left( 1 + \frac{1}{\tau_I s} \right) \]  

(4)

• Philosophy: the integral action takes into account (integrates) the past.
Ideal Proportional-Derivative (PD) Controller

- The operating output of the ideal PD controller is proportional to the weighted sum of the magnitude and the time rate of change of the error

\[ u_a(t) = K_c \left( e(t) + \tau_D \frac{de(t)}{dt} \right) + u_{ab} \]  

(5)

The derivative action is intended to anticipate future errors.

- \( \tau_D \) is the derivative time constant.

- Transfer function of the ideal PD controller

\[ G_c(s) = K_c (1 + \tau_D s) \]  

(6)

- Theoretical controller because the numerator degree of the controller transfer function \( G_c(s) \) is larger than the denominator degree; consequently, it is physically unrealizable.

- Philosophy: the derivative action takes into account (anticipates) the future.
Ideal Proportional-Integral-Derivative (PID) Controller

- Combination of P, I and D actions

\[
u_a(t) = K_c \left( e(t) + \tau_D \frac{de(t)}{dt} + \frac{1}{\tau_I} \int_0^t e(x) \, dx \right) + u_{ab} \quad (7)
\]

- Transfer function of the ideal PID controller

\[
G_c(s) = K_c \left( 1 + \tau_D s + \frac{1}{\tau_I s} \right) \quad (8)
\]

- Still derivative action is physical unrealizable.

- **Philosophy**: owing to the derivative action, the PID controller takes into account (anticipates) the future, and owing to the integral action, the PID takes into account (integrates) the past.
Comparison of P, PI and PID Controllers

- **P controller**:  
  - It presents the drawback to create a deviation of the output with respect to the set point.

- **PI controller**:  
  - It presents the advantage of eliminating the deviation between the output and the set point owing to the integral action.  
  - However, this controller can produce oscillatory responses and diminishes the closed-loop system stability.  
  - Furthermore, the integral action can become undesirable when there is saturation.

- **PID controller**:  
  - It can stabilize the output with respect to the PI controller.  
  - It is sensitive to the noise.
Sensors

- Without good measurement, it is hopeless to control the process well.
- Common sensors on chemical processes:
  - Temperature sensors: thermocouples, platinum resistance probes, pyrometers. Modelled as first- or second-order model, sometimes with a time delay.
  - Pressure sensors: classical manometers using bellows, Bourdon tube, membrane or electronic ones using strain gauges. Often represented by a second-order model.
  - Flow rate sensors: for gases, thermal mass flow meters, variable area flow meters. For liquids, turbine flow meters, depression flow meters as venturi-type flow meters, vortex flow meters, electromagnetic flow meters, sonic flow meters, Coriolis effect flow meters. Fast dynamics. Often modelled by an equation of the form

\[
\text{flow rate} = a\sqrt{\Delta P}
\]
Sensors

- Level sensors: floats, displacement, conductivity, capacitance.
- Composition sensors: potentiometers, conductimeters, chromatographs, spectrometers. Sometimes, very long delay (chromatographs).
- In the absence of a measurement concerning a given variable, if a model of the process is available, it is possible to realize a state observer called a software sensor. Used in particular for composition estimations.
- In addition of the sensor itself, there is a transmitter, simple converter and simple gain (often transmitted in the range 4-20 mA).
Actuators

- Actuators constitute material elements allowing action by means of the control loop on the process.

- Pneumatic valves.

- Design position of a valve: either completely open (fail open, or air-to-open) or completely closed (fail closed, or air-to-close) when the air pressure is not ensured.

- Many types of valves exist.

- Valve dynamics is fast.

- A valve introduces a pressure drop in the pipe.

- Valves are frequently nonlinear and can saturate.

- For liquids, flow rate $Q$ depends on the square root of the pressure drop $\Delta P_v$ of the valve

$$Q = C_v \sqrt{\frac{\Delta P}{d}}$$

(9)
**Figure**: Influence of the aperture degree of a valve on its flow rate

- **Linear (case 1)**: 
  \[ C_v = C_{vs} x \]

- **Butterfly (case 4)**: 
  \[ C_v = C_{vs} (1 - \cos\left(\frac{\pi}{2} x\right)) \]

- **Equal percentage (cases 2 and 3)**: 
  \[ C_v = C_{vs} R_v^{x-1} \]
**Figure**: Block scheme of the closed-loop process

Process closed-loop response $Y(s)$

$$Y(s) = \frac{G_p(s) G_a(s) G_c(s) K_m}{1 + G_p(s) G_a(s) G_c(s) G_m(s)} Y_r(s) + \frac{G_d(s)}{1 + G_p(s) G_a(s) G_c(s) G_m(s)} D(s)$$  \hspace{1cm} (11)
The closed-loop transfer function is equal to

\[
\frac{\text{product of the transfer functions met on the path between an input and an output}}{1 + \text{the product of all transfer functions met in the loop}}
\]

sign + is indeed the opposite of the sign of comparison between measured output and set point.

Two interesting closed loop transfer functions:
- between set point and output, influence of set point.
- between disturbance and output, influence of disturbance.
**Figure**: How the "open loop" must be understood by opposition to the closed loop.

"Open loop" and "closed loop" must be understood as electrician language.

The closed-loop transfer function is equal to:

$$1 + \text{transfer function of the open loop}$$

product of the transfer functions met on the path between an input and an output.
Dynamics of Feedback-Controlled Processes

- Closed-loop transfer function for a set point variation

\[
G_{\text{set point}} = \frac{G_p(s) G_a(s) G_c(s) K_m}{1 + G_p(s) G_a(s) G_c(s) G_m(s)}
\]  

(12)

- Closed-loop transfer function for a disturbance variation

\[
G_{\text{disturbance}} = \frac{G_d(s)}{1 + G_p(s) G_a(s) G_c(s) G_m(s)}
\]  

(13)

- Same denominators.

- Closed-loop transfer functions depend not only on the process dynamics, but also on the actuator, measurement device and the controller’s own dynamics.
Control problems

Separation between two control problems:

- Regulation (or disturbance rejection):
  - Fixed set point \( Y_r(s) = 0 \) or \( \delta y_r(t) = 0 \).
  - Process subjected to disturbances.

- Tracking:
  - Assumption that disturbance is constant \( D(s) = 0 \) or \( \delta d(t) = 0 \).
  - Set point is variable.
Study of different actions

Simplifications:

\[ G_a = 1, \ G_m = 1, \ K_m = 1 \]  \hspace{1cm} (14)

Also, assumption that the orders of process transfer function \( G_p \) and of the disturbance transfer function \( G_d \) are equal.

**Figure**: Block diagram for the study of the action of the different controllers

\[
Y(s) = \frac{G_p(s) G_c(s)}{1 + G_p(s) G_c(s)} Y_r(s) + \frac{G_d(s)}{1 + G_p(s) G_c(s)} D(s)
\]  \hspace{1cm} (15)
Proportional Action

Case of first-order systems

\[ G_p(s) = \frac{K_p}{\tau_p s + 1}; \quad G_d(s) = \frac{K_d}{\tau_d s + 1} \]  \hspace{1cm} (16)

Output

\[ Y(s) = \frac{K_p K_c}{\tau_p s + 1 + K_p K_c} Y_r(s) + \frac{K_d}{\tau_d s + 1} \frac{\tau_p s + 1}{\tau_p s + 1 + K_p K_c} D(s) \]  \hspace{1cm} (17)

Final-value theorem: \( t \to \infty \iff s \to 0 \) gives

* closed-loop steady-state gain for the set point

\[ K'_p = \frac{K_p K_c}{1 + K_p K_c} \]  \hspace{1cm} (18)

* closed-loop steady-state gain for the disturbance

\[ K'_d = \frac{K_d}{1 + K_p K_c} \]  \hspace{1cm} (19)

* \( K'_p \to 1 \) and \( K'_d \to 0 \) if \( K_c \) increases.

* Response faster in closed loop than in open loop

\[ \tau'_p = \frac{\tau_p}{1 + K_p K_c} \]  \hspace{1cm} (20)
Example of studies

1/ Tracking:
Assume step set point step of amplitude $A$

$$Y_r(s) = \frac{A}{s}; \quad D(s) = 0 \quad (21)$$

Closed-loop response to a step set point is then equal to

$$Y(s) = \frac{K_p K_c}{\tau_p s + 1 + K_p K_c} \frac{A}{s} = \frac{K'_p}{\tau'_p s + 1} \frac{A}{s} \quad (22)$$

Final value theorem

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \frac{K_p K_c}{1 + K_p K_c} A \quad (23)$$
2/ Regulation:
Assume disturbance point step of amplitude $A$

$$D(s) = \frac{A}{s}; \quad Y_r(s) = 0 \quad (24)$$

Closed-loop response to a step disturbance is then equal to

$$Y(s) = \frac{K_d}{\tau_d s + 1} \frac{\tau_p s + 1}{\tau_p s + 1 + K_p K_c} \frac{A}{s} \quad (25)$$

Final value theorem

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \frac{K_d}{1 + K_p K_c} A \quad (26)$$
Proportional Action

Case of second-order systems

\[
G_p(s) = \frac{K_p}{\tau_p^2 s^2 + 2 \zeta_p \tau_p s + 1}; \quad G_d(s) = \frac{K_d}{\tau_d^2 s^2 + 2 \zeta_d \tau_d s + 1}
\] (27)

Output \(Y(s)\)

\[
Y(s) = \frac{K_p K_c}{\tau_p^2 s^2 + 2 \zeta_p \tau_p s + 1 + K_p K_c} Y_r(s) + \frac{K_d}{\tau_d^2 s^2 + 2 \zeta_d \tau_d s + 1} D(s)
\] (28)
1/ Tracking
Modification of period and damping factor

\[
\tau'_p = \frac{\tau_p}{\sqrt{1 + K_p K_c}} \quad \zeta'_p = \frac{\zeta_p}{\sqrt{1 + K_p K_c}} \quad \text{(29)}
\]

Steady-state gain

\[
K'_p = \frac{K_p K_c}{1 + K_p K_c} \quad \text{(30)}
\]

Same remarks as for first-order. A deviation between the set point and the asymptotic response exists which is all the more important as the gain is low.

2/ Regulation
Same remarks as for first-order.
Integral Action

Even if integral action is never used alone, in order to characterize its influence, we first assume that the controller is pure integral

\[ G_c(s) = \frac{K_c}{\tau_I s} \]  

(31)

Study of first-order systems.

\[ Y(s) = \frac{K_p}{1 + \tau_p s} \frac{K_c}{\tau_I s} Y_r(s) + \frac{K_d}{1 + \tau_d s} \frac{K_c}{\tau_I s} D(s) \]  

(32)

or

\[ Y(s) = \frac{1}{\tau_p \tau_I} \left( \frac{1}{s^2 + \frac{\tau_I}{K_p K_c}} + \frac{\tau_I}{K_p K_c} \right) Y_r(s) + \frac{K_d}{\tau_d s + 1} \frac{\tau_I s}{\tau_I \tau_p s^2 + \tau_I s + K_p K_c} D(s) \]  

(33)

Modification of the order in closed loop.
Integral Action

\[ Y(s) = \frac{1}{\tau_p \tau_I \frac{s^2}{K_p K_c} + \tau_I \frac{s}{K_p K_c}} Y_r(s) + \frac{K_d}{\tau_d s + 1} \frac{(\tau_p s + 1) \tau_I s}{\tau_I \tau_p s^2 + \tau_I s + K_p K_c} D(s) \quad (34) \]

1/ Tracking, step set point of amplitude \( A \)

\[ Y_r(s) = \frac{A}{s} \quad (35) \]

hence

\[ Y(s) = \frac{1}{\tau_p \tau_I \frac{s^2}{K_p K_c} + \tau_I \frac{s}{K_p K_c}} \frac{A}{s} \quad (36) \]

Final value theorem

\[ \lim_{t \to \infty} y(t) = \lim_{s \to 0} [s \cdot Y(s)] = A \quad (37) \]

The integral action eliminates the asymptotic deviation. Response faster when gain is high, but oscillatory responses.
Integral Action

\[ Y(s) = \frac{1}{\tau_p \tau_l s^2 + \tau_l s + 1} \left( Y_r(s) + \frac{K_d}{\tau_d s + 1} \frac{(\tau_p s + 1) \tau_l s}{\tau_l \tau_p s^2 + \tau_l s + K_p K_c} D(s) \right) \]  

(38)

2/ Regulation, step disturbance of amplitude \( A \)

\[ D(s) = \frac{A}{s} \]  

(39)

hence

\[ Y(s) = \frac{K_d}{\tau_d s + 1} \frac{(\tau_p s + 1) \tau_l s}{\tau_l \tau_p s^2 + \tau_l s + K_p K_c} \frac{A}{s} \]  

(40)

Final value theorem

\[ \lim_{t \to \infty} y(t) = \lim_{s \to 0} [s \ Y(s)] = 0 \]  

(41)

The integral action rejects step-like disturbances.
Transfer function of a pure ideal derivative controller is equal to

\[ G_c(s) = K_c \tau_D s \]  \hspace{1cm} (42)

Improper transfer function (numerator degree larger than denominator degree).

Block diagram of the ideal PID controller

Block diagram of a real PID controller
Ideal Derivative Action

Response \( Y(s) \) to a set point or disturbance variation

\[
Y(s) = \frac{K_p}{\tau_p s + 1} \frac{K_c}{K_p} \frac{\tau_D s}{s} Y_r(s) + \frac{K_d}{\tau_d s + 1} D(s)
\]

\[
= \frac{K_p}{(\tau_p + K_p K_c \tau_D) s + 1} Y_r(s) + \frac{K_d}{\tau_d s + 1} \frac{\tau_p s + 1}{(\tau_p + K_p K_c \tau_D) s + 1} D(s)
\]

No influence on the system order.

Closed-loop time constant

\[
\tau'_p = \tau_p + K_p K_c \tau_D
\]

Closed-loop response will be slower than the open-loop one.

Stabilization of the process.
PI Controller

Transfer function of PI Controller

\[ G_c(s) = K_c \left( 1 + \frac{1}{\tau_I s} \right) \] (45)

Output

\[ Y(s) = \frac{\tau_I s + 1}{\tau_d s + 1} Y_r(s) + \frac{\tau_d s + 1}{\tau_p \tau_I s^2 + \tau_I} \left( K_p K_c + 1 \right) s + 1 \]

or

\[ Y(s) = \frac{K_d}{\tau_d s + 1} \frac{\tau_d s + 1}{\tau_p \tau_I s^2 + \tau_I} \left( K_p K_c + 1 \right) s + 1 \]

Conclusions for the Integral action are still valid.
Summary of Controller Characteristics

- **P controller**:  
  - one tuning parameter $K_r$,  
  - deviation from the set point, decreased by increasing gain.

- **PI controller**:  
  - elimination of the deviation between the asymptotic state and the set point,  
  - faster response, possible oscillations,  
  - possible saturation of control variable $u$.

- **PID controller**:  
  - same interest as PI,  
  - stabilizing effect,  
  - sensitivity to measurement noise.
Influence of controller on first-order system

Response of a first-order system \((K_p = 5, \tau_p = 10)\) to a set point unit step.

P controller

\[ K_c = 2 \]
\[ \tau_I = 20 \]

PI controller

\[ K_c = 2 \]
\[ \tau_I = 20 \]

Real PID controller

\[ K_c = 2 \]
\[ \tau_I = 20 \]
\[ \tau_D = 1 \]
\[ \beta = 0.1 \]
Influence of controller on second-order system

response of a second-order system \((K_p = 5, \tau_p = 10, \zeta_p = 0.5)\) to a set point unit step.

P controller

\[ K_c = 2 \]

\[ \tau_I = 20 \]

PI controller

\[ K_c = 2 \]

\[ \tau_I = 20 \]

Real PID controller

\[ K_c = 2 \]

\[ \tau_I = 20 \]

\[ \tau_D = 1 \]

\[ \beta = 0.1 \]
Influence of controller on first-order system

Response of a first-order system \((K_p = 5, \tau_p = 10 : K_d = 2, \tau_d = 2)\) to a disturbance unit step.

**P controller**

**PI controller**

**Real PID controller**

\[K_c = 2\]

\[\tau_I = 20\]

\[\tau_D = 1\]

\[\beta = 0.1\]
Influence of controller on second-order system

Response of a second-order system

\( (K_p = 5, \tau_p = 10, \zeta_p = 0.5 : K_d = 2, \tau_d = 2, \zeta_d = 0.25) \) to a disturbance unit step.

### P controller

\[ K_c = 2 \]

\[ \tau_I = 20 \]

### PI controller

\[ K_c = 2 \]

\[ \tau_I = 20 \]

### Real PID controller

\[ K_c = 2 \]

\[ \tau_D = 1 \]

\[ \beta = 0.1 \]