Geodesic acoustic mode excitation by a spatially broad energetic particle beam

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Global radial eigenmodes of energetic-particle-induced geodesic acoustic mode are systematically studied, and their properties are found to depend on the nonuniformities of both the geodesic acoustic mode continuous spectrum and the energetic particle (EP) radial density profile. For a broad EP drive, the eigenmode equation valid for arbitrary EP drift orbit width is derived to study the broad EP density and/or GAM continuum profiles is discussed in several recent works. Here, we remind two important scale continuum, a model dispersion relation of global EGAM was obtained, demonstrating the finite drive threshold of EGAM due to the GAM continuum damping. The excitation of EGAM by a radially localized EP beam was then investigated in Ref. 10. With the energetic particle beam localized away from the position where the mode frequency matches that of the GAM continuum in order to minimize the continuum damping, the obtained global EGAM radial mode structure shows that EGAM is self trapped by the localized EP beam, with an exponentially small tunneling coupling to propagating kinetic GAM (KGAM), resulting in an exponentially small EGAM excitation threshold.

However, in realistic tokamak plasmas, the EP beam radial density profile has a typical scale length comparable with the characteristic scale length of GAM continuous spectrum; thus, the excited EGAM is expected to be strongly dependent on the radial structures due to radial profiles of both EP beam and GAM continuum. In this work, we will extend the theory of EGAM to the case with a broad EP drive. This paper is organized as follows: In Sec. II, the dependence of the EGAM radial mode structure on EP radial density profile and GAM continuum is discussed; in Sec. III, an eigenmode equation that is valid for arbitrary EP drift orbit width is derived to study the broad EGAM problem, which is the relevant case for typical experimental condition; then, the broad EGAM eigenmode equation is solved numerically in Sec. IV to give mode structure and dispersion relation of global EGAM. Finally, conclusions and discussions are presented in Sec. V.

II. DEPENDENCE OF EGAM RADIAL STRUCTURE ON EP AND GAM CONTINUUM RADIAL PROFILES

The dependence of the EGAM radial structure on EP density and/or GAM continuum profiles is discussed in several recent works. Here, we remind two important scale
lengths that determine the nonlocal properties of GAM/EGAM problem: the characteristic scale length of GAM continuous spectrum $L_G \equiv \left| \omega_G^2(r)/(\partial \omega_G^2(r)/\partial r) \right|$ and the scale length of $E$-profile $L_E \equiv \left| n_E(r)/(\partial n_E(r)/\partial r) \right|$. Here, $\omega_G$ and $n_E$ are, respectively, the GAM continuum frequency and $E$ profile density. It is pointed out in Refs. 9, 10, and 12 that it is the relative ordering between $L_E$ and $L_G$ that determines the fundamental nonlocal properties of GAM/EGAM, which can be readily seen from the EGAM eigenmode equation \(^{10,9,9}\)

$$-\frac{e}{m} n_e \frac{1}{\Omega_e^2} \frac{\partial}{\partial r} \delta \phi \frac{\partial}{\partial r} \delta \phi + \delta n_\phi = 0.$$  \tag{1}$$

Here, $e$ is the unit charge, $m$ is the ion mass, $n_e$ is the equilibrium core ion density, $\Omega_e$ is the ion cyclotron frequency, $\delta \phi$ is the perturbed electrostatic potential, $\delta n_\phi$ represents the surface averaging, and $\delta \phi_c$ is the GAM dispersion relation, reading

$$\delta \phi_c = -1 + \frac{\omega_0^2(r)}{\omega_0^2} - \frac{G}{2} \delta \phi_c \frac{\partial^2}{\partial r^2},$$  \tag{2}$$

with $k$ being the radial wavenumber, $\rho_L(r)$ the thermal ion Larmor radius and the expression of $G$, derived in Ref. 10, is given by

$$G = (747/32 + 481 \tau/32 + 35 \tau^2/8 + \tau^3/2) \nu_i^3/(R_0^2 \omega_0^2) - (13/4 + 3\tau + \tau^2) \nu_i^3/(R_0^2 \omega_0^2) + 3/4.\text{In Eq. (1), the first term is the surface averaged core plasma density perturbation, reflecting the singularity due to GAM continuum at } \delta \phi_c = 0. \text{The second term of Eq. (1), } \delta n_\phi, \text{is the surface averaged perturbed EP density, which is an integral operator, in the form}

$$\delta n_\phi = 2\pi B \sum_{\sigma = \pm 1} \left[ \frac{\left( EdA \delta E \right)}{|v_i|} \frac{\partial F_{oh}}{|m\partial E|} \delta \phi + J_0(k_{\perp} \rho_{L,h}) \delta H_{b} \right],$$  \tag{3}$$

where $E = (v_i^2 + v_T^2)/2$, $\Lambda \equiv \mu/E$ is the pitch angle, $\sigma = 1/|v_i|$, $\mu = v_i^2/(2B)$ is the magnetic moment, $F_{oh}$ is the EP equilibrium distribution function, and $\delta H_b$ is the EP nonadiabatic response, which can be obtained from the linear gyrokinetic equation \(^{15,16}\)

$$\left( \omega - \omega_d + i \omega_{te} \frac{\partial}{\partial \theta} \right) \delta H = -\frac{e}{m} F_0 \frac{\partial}{\partial r} J_0(k_{\perp} \rho_{L,h}) \delta \phi.$$  \tag{4}$$

Here, $\omega_{te} = |v_i|/q R_0$ is the transit frequency, $q$ is the safety factor, $\omega_d = \omega_0 \sin \theta = -k_{\perp} (v_i^2 + 2v_T^2)/(2\Omega_0 R_0) \sin \theta$ is the magnetic drift frequency associated with the geodesic curvature, $k_{\perp}$ is the perpendicular wave-vector, $k_\perp \equiv k_\perp \equiv -i/(\partial/\partial r)$ is an operator, $\rho_{L,h}$ stands for the EP Larmor radius, $J_0$ is the Bessel function accounting for finite Larmor radius (FLR) effects, and $\theta$ is the poloidal angle-like coordinate of the torus with $(r, \theta, \xi)$ being straight-field-line toroidal flux coordinates. In order to make further analytical progress, we need to simplify Eq. (3), which is in general an integral equation, reducing it to a differential equation. Such reduced forms have been obtained already in several approximate limits.\(^{5,7,9,10}\) Here, we will extend this treatment in order to investigate the experimental relevant case of a broad EP beam.

The dependence of the EGAM radial structure on the $E$ profile density is first discussed in Ref. 5, where a radially localized $E$ profile is assumed and a hybrid model is used; thus, background thermal plasma is described by MHD fluid equations and EPs are described by kinetic equations; meanwhile the coupling to GAM continuum is neglected. In this case, assuming the ordering $\rho_{L,b} \ll L_E$, the EP response to the perturbed electrostatic potential can be calculated for small EP drift orbits $(k_{\perp} \rho_{L,h} \ll 1)$, with $\rho_{L,h} \approx q R_{E,b}$ being the EP drift orbit and $\rho_{L,h}$ being the EP Larmor radius. In the small drift orbit limit, the integral operator for surface averaged EP perturbed density, $\delta n_\phi$, can be approximated as \(^{5,7,9,10}\)

$$\delta n_{\phi}(k_{\perp} \rho_d \ll 1) = -\frac{e}{m} n_e (\varphi) \delta \phi_c \int \delta n_{\phi} \frac{1}{\Omega_e^2} \left[ \frac{e}{m} F_0 \frac{\partial}{\partial r} \frac{\partial^2}{\partial r^2} \right].$$  \tag{5}$$

For a single pitch angle slowing down EP equilibrium distribution function assumed in Ref. 10, i.e., $F_{oh} = \sqrt{2} (1 - \Lambda_0 B) n_E(r)(\lambda - \Lambda_0)\Theta(1 - E/E_b)/(4\pi B \ln(E_b/E_c) (E_b^2/E_c^2)),$

$$\delta n_{\phi} \approx \frac{(2 - \Lambda_0)(-2 + 5\Lambda_0)}{2(1 - \Lambda_0 B)^{3/2}} \ln\left( 1 - \frac{\omega_{e,b}^2}{\omega_0^2} \right) + \Lambda_0 B (2 - \Lambda_0 B)^2 \left( 1 - \frac{\omega_{e,b}^2}{\omega_0^2} \right),$$  \tag{6}$$

where $\rho_{L,b} = q \sqrt{2E_{b}/\Omega_e}$, $\omega_{e,b} = \sqrt{2E_{b} (1 - \Lambda_0 R_0)}/(q R_0)$, and $H = (2 - \Lambda_0 B)^2 \left[ 3(2\Lambda_0 B^2 + 5\Lambda_0 B - 2)/8 - (\Lambda_0 B (2 - \Lambda_0 B)(1 - \omega_{e,b}^2)/\omega_0^2) + (9\Lambda_0 B - 2)\omega_0^2 \ln\left( 1 - \omega_{e,b}^2/\omega_0^2 \right) / (2\omega_{e,b}^2) \right] + (\Lambda_0 B (2 - \Lambda_0 B)(1 - 4\omega_{e,b}^2/\omega_0^2) + (9\Lambda_0 B - 2)\omega_0^2 \ln\left( 1 - 4\omega_{e,b}^2/\omega_0^2 \right) / (2(1 - \Lambda_0 B)^{3/2})].$

Meanwhile, the normalized EP density (to core plasma density) is $n_b = \sqrt{1 - \Lambda_0 B^2 n_{E,R}}(4\ln(E_b/E_c)/E_c)$, with $E_b$ and $E_c$ being, respectively, the birth and critical energy of a slowing down distribution, and $\Theta(1 - E/E_b)$ is the Heaviside step function. The localized EP drive creates a potential well and traps the excited EGAM eigenmode, with the characteristic wavelength being the geometric mean of EP drift orbit $\rho_{L,b}$ and the EP density profile scale length $L_E$.\(^{10,17}\) This result also justifies the $k_{\perp} \rho_{L,b} \ll 1$ assumption \textit{a posteriori}.\(^{10}\)

When the background thermal plasma is treated by MHD fluid theory,\(^{12}\) kinetic mode dispersiveness is only due to EP finite drift orbit width effect (FOW); thus, the mode cannot propagate in the region where EP density (drive) is negligible and the EGAM eigenmode described by this model can be localized inside the EP density profile only. EGAM driven by a sharply radially localized EP source with $L_E \ll L_0$ is also considered in Ref. 10; however, both thermal ions and EPs are treated by kinetic theories and coupling to GAM continuum is analyzed as well.

When coupling to GAM continuum is taken into account, the mode structure becomes singular at the resonance point between mode and local GAM continuum
frequency. Thus, the EP response in this region is characterized by large drift orbits, i.e., \( k_r \rho_{d,b} \gg 1 \). In the large drift orbit limit, the EP response will be mainly adiabatic, with a finite residual resonance drive. This generally introduces an additional term to the EGAM eigenmode equation, which can be formally written as

\[
\frac{\partial}{\partial r} (E_r + \delta E_h) \frac{\partial \delta \phi}{\partial r} + n_E(r_c)\delta E_h \frac{\partial^2 \delta \phi}{\partial \phi^2} = 0. \tag{7}
\]

Here, \( \delta E_h \) and \( \delta \phi \) are, respectively, the residual resonance drive and the adiabatic response at the resonance layer with GAM continuum. These two terms are, to the lowest order, independent of \( k_r \). Note that the last term in Eq. (7) is proportional to EP density at \( r_c \) where EGAM is coupled to the GAM continuum. So, if we take the EP beam radially localized away from the position where GAM continuum frequency is the same as the mode frequency (EP characteristic frequency), the additional term is negligible due to the vanishing EP density at the resonant point. Thus, consistent frequency, the additional term is negligible due to the vanishing EP density at the resonant point. Thus, consistent with the EP FOW effect inside the EP localization domain, while the thermal ion FLR/FOW takes over when EP density is vanishingly small outside the EP localization domain. When the FLR and FOW effects of thermal ions are properly treated in the region where EP density is vanishingly small outside the EP localization domain, the additional term to the EGAM eigenmode equation, which is valid for arbitrary EP drift orbit widths, can effectively drive the EGAM mode, the EP response is characterized by small drift orbit widths; in the resonance layer, the EP response is characterized by large drift orbit widths and EPs respond adiabatically to the fluctuating potential. The wavelength, thus, varies continuously across the regions and, to correctly understand the problem of EGAM driven by a broad EP source, we need an eigenmode equation that is valid for arbitrary EP drift orbit widths. The simplest approximation is based on construction of a Padé approximation of the EP response \( \overline{\delta E_h} \) based on its expression in the small and large drift orbit limits. This is carried out in Sec. III.

### III. PADÉ APPROXIMATION EIGENMODE EQUATION FOR EGAM DRIVEN BY A BROAD EP SOURCE

The EP response in the small orbit limit is already discussed in several papers, and, for a single pitch angle slowing down equilibrium EP distribution function, is given by Eq. (5). Here, we will derive the EP response for the same EP distribution function in the large drift orbit limit, where the EP nonadiabatic response is suppressed by the large drift orbit width relative to the mode wavelength. This can be readily seen by comparing the two terms of Eq. (3), which are, respectively, the adiabatic and the nonadiabatic response. Noting the asymptotic form of \( J_0(k_r \rho_{d,b}) \propto 1/\sqrt{k_r \rho_{d,b}} \), the nonadiabatic term is \( O(1/k_r \rho_{d,b}) \) smaller than the adiabatic term and, for the single pitch angle slowing down equilibrium EP distribution function, the surface averaged perturbed EP density can be readily calculated as

\[
\overline{\delta n_b}(k_r \rho_{d,b} \gg 1) = \frac{e n_b(r) \delta \phi}{\Omega_i^2} \delta E_h, \tag{8}
\]

in which,

\[
\delta E_h = \frac{1}{3(1 - \Lambda_0 B_0) E_b \ln(E_b/E_r)} \left[ \arctan \left( \frac{2 E_{b,1/2}}{3 E_c^{1/2}} \right) + \arctan \left( \frac{1}{3} \right) \right]. \tag{9}
\]

We can then construct the Padé approximation EP response based on Eqs. (5) and (8), which in the simplest form that properly reproduces EP responses in the limiting cases, is given by

\[
\overline{\delta n_b} = \frac{e n_b N_i \delta \phi}{m \Omega_i^2} k_r^2 \left( \delta \phi + \frac{H k_c^2 \rho_{d,b}^2}{1 + k_r^2 \rho_{d,b}^2 N_i N_b H} \right) \frac{1}{2 \pi^2} V(k_r), \tag{10}
\]

This Padé approximation, as shown in Fig. 1, smoothly connects the EP response in small and large drift orbit widths limits and qualitatively describes the EP response as \( k_r \rho_{d,b} \) varies. We note, here, that the equivalent potential function \( V(k_r) \) is independent of \( r \).

With the Padé approximation of the EP response, the EGAM eigenmode equation (1) becomes a sixth order differential equation in real space, which is difficult to solve. However, if we take a linear radial dependence of the GAM dielectric function, \( \delta \phi \simeq \delta \phi_0 (1 - (r - r_0)/L_b) \), and assume a Lorentzian distribution for the EP radial density profile, \( n_E(r) = n_E(r_0)/(1 + (r - r_0)^2/L_b^2) \), Eq. (1) becomes a 3rd order differential equation in the Fourier space, i.e.,

\[
\left[ \delta \phi_0 \left( \frac{i}{L_b \partial k_r} - 1 \right) \left( \frac{\partial^2}{L_b^2 \partial k_r^2} - 1 \right) - \frac{G \rho_{d,b}^2}{2} \left( \frac{\partial^2}{L_b^2 \partial k_r^2} - 1 \right) k_r^2 + N_b(r_0) V(k_r) \right] \delta E_r = 0. \tag{11}
\]
The second term of Eq. (11), which comes from the FOW/FLR of thermal ions and describes the underlying physics of kinetic damping as mode conversion to the short wavelength kinetic wave,\textsuperscript{20–22,24} is needed in real space mode equation to remove the singularity, since the coefficient of the highest order radial derivative term vanishes at the resonant point.\textsuperscript{22,23} However, in Fourier space, since Eq. (11) is regular, we can drop this term with the reduced description describing convective (continuum) damping instead of mode conversion.\textsuperscript{26} Hence, in the rest of this work, we will treat the following reduced Padé approximation EGAM eigenmode equation:

\[
\delta E_r \left( \frac{i}{L_G} \frac{\partial}{\partial k_r} - 1 \right) \left( \frac{\partial^2}{L_G^2 \partial k_r^2} - 1 \right) + N_b(r_0) V(k_r) \right] \delta E_r = 0.
\]

(12)

As \(|k_r\rho_{i,\beta}| \rightarrow \infty\), \(V(k_r)\) vanishes as \(O(1/k_r^2)\), corresponding to the fact that EPs do not contribute to the inertial layer physics due to the large drift orbit width compared with the mode wavelength; thus, the inertial layer contribution comes only from thermal plasma. The reduced Padé EGAM eigenmode equation has the following (out-going wave) boundary condition as \(|k_r| \rightarrow \infty\):

\[
\delta E_r(k_r \rightarrow +\infty) = a \exp(-L_0 k_r) + b \exp(-iL_G k_r),
\]

(13)

\[
\delta E_r(k_r \rightarrow -\infty) = c \exp(L_0 k_r).
\]

(14)

As \(|k_r| \rightarrow \infty\), the two exponentially decay terms reflect the fact that EGAM cannot be effectively driven at small radial scales; while the \(\exp(-iL_G k_r)\) term, with a positive (outward) “group velocity” in Fourier space, generates singular radial mode structures at the resonant point with GAM continuum, resulting into finite continuum damping. If the thermal ion FLR/FOW effect is properly taken into account, it creates an additional potential well\textsuperscript{25} and prevents the mode structure in Fourier space to propagate into regions with \(|k_r| \gg \rho_{Li}^{1/3} L_G^{1/3}\). This effect, of course, corresponds to resolving the singularity in real space and describes thus mode conversion to kinetic GAM.\textsuperscript{20,21}

The analytic dispersion relation of the reduced Padé EGAM eigenmode equation (12) can be formally derived as variational principle. That is, multiplying \(\delta E_r^*\) to Eq. (12) and subtracting its complex conjugate, then integrating over the Fourier space, we get

\[
-2i \int_{-\infty}^{\infty} N_b(r_0) \frac{\partial \text{Re}(V(k_r)/\delta_{r,0})}{\partial k_r} \delta E_r^* \, dk_r = \int_{-\infty}^{\infty} N_b(r_0) \text{Im}(V(k_r)/\delta_{r,0}) \delta E_r^2 \, dk_r
\]

\[
+ \left[ \frac{i}{L_G L_B} \left( \delta E_r^* \frac{\partial^2 \delta E_r}{\partial k_r^2} + \delta E_r \frac{\partial^2 \delta E_r^*}{\partial k_r^2} - \frac{|\delta E_r|^2}{\partial k_r^2} \right) \right]
\]

\[
- \frac{1}{L_B} \left( \frac{\partial \delta E_r}{\partial k_r} - \frac{\partial \delta E_r^*}{\partial k_r} - i \frac{\partial |\delta E_r|^2}{\partial k_r} \right) \right|_{k_r = -\infty}^{k_r = \infty}.
\]

(15)

In Eq. (15), the left hand side represents the rate of change of the total energy and \(\gamma\) is the imaginary part of eigenmode frequency \(\omega\). On the right hand side, the first term represents the EP resonant drive, while the term in square brackets represents dissipation due to generation of short wavelength structures. We note that, as is evident from Eq. (8), the EP response is purely real in the large orbit limit, so EP drive is dominated by the small-\(k_r\) region. The second term ((\(\delta E_r^* \delta_{r,0} \delta E_r - \delta E_r \delta_{r,0} \delta E_r^* \)/\(L_B^2\)) of the dissipation response, meanwhile, has the familiar form of the probability current for Schrödinger’s equation. In fact, the square bracket is the “generalized probability current” of the third order differential equation. Thus, Eq. (15) describes the threshold for EGAM excitation due to continuum damping: The EP drive in the ideal region must exceed the continuum damping due to coupling to the GAM continuous spectrum in the inertial region for the EGAM instability. Applying the boundary conditions (13) and (14), we get the formal dispersion relation of the global EGAM:

\[
\gamma \int_{-\infty}^{\infty} N_b(r_0) \frac{\partial \text{Re}(V(k_r)/\delta_{r,0})}{\partial k_r} \delta E_r^* \, dk_r
\]

\[
= \int_{-\infty}^{\infty} N_b(r_0) \text{Im}(V(k_r)/\delta_{r,0}) \delta E_r^2 \, dk_r + \frac{(L_B^2 + L_G^2) b^2}{2 L_G L_B}. \gamma \int_{-\infty}^{\infty} N_b(r_0) \frac{\partial \text{Re}(V(k_r)/\delta_{r,0})}{\partial k_r} \delta E_r^* \, dk_r
\]

(16)

Note that the left hand side has a negative sign, and the first term on the right hand side represents the EP resonance drive while the second term represents continuum damping. In Eq. (16), the coefficient “\(b^2\)” corresponds to the ratio of the mode amplitude at the resonant point comparing to that at the center of EP localization, and is to be determined from numerical solution of the reduced EGAM eigenmode equation.

We note that Eq. (16) is exactly the Fourier space counterpart of Eq. (11) of Ref. 9, and the dissipation due to propagating to large-\(k_r\) region corresponds exactly to the singular absorption in Ref. 9. In fact, this is evident from the inverse
Fourier transformation of the second term in the boundary condition (13).

IV. NUMERICAL SOLUTION OF THE PADÉ EGAM EIGENMODE EQUATION

As a third-order ordinary differential equation, the reduced EGAM eigenmode equation can be solved numerically by the shooting method. Here, we take the EP radial profile scale $L_\theta = 2$, the normalized (to GAM continuum frequency at the center of EP localization $r_0$) EP transit frequency $\omega_{tr, b} = 1$, the EP drift orbit $\rho_{d, b} = 0.5$, $H = 1$, the normalized EP peak density $N_b = 0.05$, while we vary the GAM continuum scale length $L_G$ from 1.5 to 5 to study the EP threshold due to coupling to GAM continuum. The corresponding eigenmode structure in Fourier space for $L_G = 3.5$ is shown in the left panel of Fig. 2, in which the solid curve and the dashed curve are, respectively, the real and imaginary parts. We note that, even though the obtained eigenmode structure looks very similar to the EGAM eigenmode structure obtained in Ref. 10 (Fig. 4 therein), the physics meaning is completely different, in that what we have here is the “eigenmode structure” in Fourier space for a radially broad EP drive, while what is shown in Ref. 10 is the eigenmode structure in real space for a radially localized EP drive. We observe that the mode is predominantly localized in the small-$k_r$ region, where the EP drive is most effective. Meanwhile, the outward-propagating tail in the $k_r$ space, as noted in Eq. (16), corresponds to the generation of short radial-wavelength structures at the resonant point with the GAM continuum, as expected from Eq. (13) and shown in the right panel of Fig. 2. This “outward” propagating tail, and thus, the coupling of EGAM to GAM continuum, increases with decreasing $L_G/L_\theta$. When $L_G$ is comparable with $L_\theta$, the EGAM suffers heavy continuum damping due to strong coupling to GAM continuum, as illustrated in the right panel of Fig. 2, where the second peak at $x \approx 3.5$ shows the corresponding singular mode structure in real space. This resonant interaction with GAM continuum determines the EP density threshold for EGAM instability. The dependence of global EGAM growth rate on the normalized EP density is shown in Fig. 3. The EP density profile scale length is fixed as $L_\theta = 2$, and we decrease the GAM continuum scale length $L_G$ from 5 to 1.5. We can see that a clear threshold appears when $L_G$ reaches 1.5. This corresponds to a case with strong EGAM coupling to the GAM continuum.

The mode structure in real space, as shown in the right panel of Fig. 2, is obtained from inverse Fourier transformation of the mode structure in Fourier-$k_r$ space. The secondary peak in the mode structure at $x \approx 3.5$ shows the corresponding rapidly varying radial dependence in real space caused by the singularity in GAM continuum at $x = 3.5$. The peak
will be more singular if we use the EGAM eigenmode structure in a broader $k_r$ region to increase the resolution in real space and, simultaneously, if we make the mode drive weaker. In the present case, the sharpness of the mode structure is limited by the finite growth rate of EGAM, which gives the mode frequency (and the mode structure at the resonance point with GAM continuum) an uncertainty of the order of $\gamma$. If we were in the near marginal instability limit, with a very small growth rate $\gamma \ll \omega_r$, the corresponding mode structure in the singular layer of GAM continuum will be really “singular”; this is the case discussed in a previous work\textsuperscript{10} (the mode structure in real space is shown in Fig. 4 therein). However, we note that, in Ref. 10, the singularity is removed by finite FLR/FOW effects of thermal ions (and the corresponding radial structure is broadened), while in this paper, we formally keep the singularity by ignoring the thermal ion FLR/FOW effects, as we discussed after Eq. (11), but the mode structure is broadened by the finite growth rate.

V. CONCLUSIONS AND DISCUSSIONS

In the present work, we have extended the previous theoretical analysis\textsuperscript{5,9,10} of the EGAM stability with weak coupling to GAM continuum to the case where the EP profile is sufficiently broad to strongly affect global mode structure and coupling to the GAM continuum. Due to the existence of multiple spatial scales, from macroscopic scales of equilibrium profiles, to the mesoscopic scales of EGAM localization width, and then to the microscopic scale of resonant mode-converted KGAM, one, in principle, needs to solve the global eigenmode stability properties in terms of an integral equation. Thus, in order to make analytic progress, we have made the simplifying Padé approximation to reduce the eigenmode equation to a 6th order ordinary differential equation; which can be Fourier transformed to a 3rd-order ordinary differential equation in the Fourier $k_r$ space. Both analytical and numerical analyses have been carried out using this eigenmode equation in the $k_r$ space; demonstrating that, when the EP scale length is comparable to that of GAM, a finite threshold of $n_p/n_e$ exists to excite the EGAM instability. Such threshold is due to the strong coupling to GAM continuum and the resultant generation of “singular” radial structures and associated resonant continuum damping.

The difficulty in dealing with the broad EP beam case lies in that, due to the cross-scale coupling induced by the resonant coupling of EGAM and GAM continuum, the EGAM eigenmode equation is in general an integro-differential equation. In the present work, we simplify the analysis by first introducing the Padé approximation and then solving the corresponding eigenmode equation in the Fourier space. It will be illuminating to extend the Fourier transformation technique to the case where, instead of the Padé approximation, $\delta n_t$ is given by the more exact response; i.e., Eq. (3). Investigation employing this approach is currently under study, and the results will be reported in a future publication.

The cross-scale coupling, extending from micro- to meso- to macro-scales in the case of the global EGAM problem, investigated in this work, is mediated by the unique role played by EPs. In this respect, the problem under investigation encompasses many peculiar features of expected behaviors of burning plasmas of fusion interest. This makes the extension of the present work to the nonlinear dynamics aspects even more interesting and of great potential impact in the broader sense.

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