Communication-efficient distributed strategy for reactive power optimisation considering the uncertainty of renewable generation

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Abstract: Due to the non-linear power flow equality constraints and uncertain injected power of renewable generation, the reactive power optimisation problem is stochastic and non-convex, which is hard to be solved efficiently. Therefore, the discrete probability model of renewable generation is utilised to build the multi-scenario deterministic formulation of the stochastic problem, which is further changed to a conic optimisation model via the second-order conic relaxation. By using the skill of splitting buses and the augmented Lagrangian decomposition, the problem is separated into several smaller manageable sub-problems. Moreover, on the basis of the accelerated local augmented Lagrangian, a fully distributed conic optimisation algorithm is developed and the global optimal solution can be obtained iteratively in a parallel fashion. During the iterative procedure of the algorithm, only partial local message, without central coordination, need to be exchanged between the neighbouring subsystems. Each subsystem is interested in knowing only some components rather than the entire information, which improves communication efficiency. Simulations verify the effectiveness of the proposed algorithm.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$E$, $\Psi$</td>
<td>bus set and branch set of the distribution system</td>
</tr>
<tr>
<td>$E_i$, $\Psi_i$</td>
<td>nodal set and edge set of the communication network</td>
</tr>
<tr>
<td>$D$</td>
<td>feasible set of whole system</td>
</tr>
<tr>
<td>$D_k$</td>
<td>feasible set of subsystem $k$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>set of scenarios</td>
</tr>
<tr>
<td>$Y_{ij}$</td>
<td>probability set of the corresponding discretised states of renewable power at $i$th bus</td>
</tr>
<tr>
<td>$E_{nk}$</td>
<td>set of neighbours of subsystem $k$</td>
</tr>
<tr>
<td>$E_{kG}$</td>
<td>set of textireceiving subsystems of subsystem $k$</td>
</tr>
<tr>
<td>$E_{kG}$</td>
<td>set of textisending subsystems of subsystem $k$</td>
</tr>
<tr>
<td>$\gamma_{ij}$</td>
<td>discrete states set of renewable power at $i$th bus</td>
</tr>
<tr>
<td>$\rho_{i\theta}$</td>
<td>probability of scenario $\theta$</td>
</tr>
<tr>
<td>$I_j$</td>
<td>complex current flowing through the $j$th branch</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>lower and upper bounds of branch current</td>
</tr>
<tr>
<td>$P_p$, $Q_i$</td>
<td>sending-end active and reactive powers of the $j$th branch</td>
</tr>
<tr>
<td>$p_i^D$, $q_i^D$</td>
<td>active and reactive demand powers at the $j$th bus</td>
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<td>$p_i^G$, $q_i^G$</td>
<td>injected active and reactive powers of generators at the $j$th bus</td>
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<tr>
<td>$P_{Wij}$, $P_{ij}$</td>
<td>wind power and solar power injected at the $j$th bus</td>
</tr>
<tr>
<td>$P_{Wmax}$, $P_{i max}$</td>
<td>maximum power of WTs and PV arrays at $j$th bus</td>
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<tr>
<td>$q_{i max}$, $q_{i max}$</td>
<td>lower and upper bounds of active power at $j$th bus</td>
</tr>
<tr>
<td>$R_j$, $Y_j$</td>
<td>resistance and reactance of the $j$th branch</td>
</tr>
<tr>
<td>$V_j$</td>
<td>complex voltage at $j$th bus</td>
</tr>
<tr>
<td>$V_{min}$, $V_{max}$</td>
<td>lower and upper bounds of bus voltage</td>
</tr>
<tr>
<td>$Y_a$</td>
<td>optimal variable of the whole distribution system</td>
</tr>
<tr>
<td>$z_k$</td>
<td>core variable of the subsystem $a$</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Lagrangian multipliers</td>
</tr>
<tr>
<td>$\Delta^1_{ij}$, $\Delta^2_{ij}$</td>
<td>residual errors at $i$th iteration</td>
</tr>
<tr>
<td>$c_{un}$, $k_{we}$</td>
<td>parameters of Weibull distribution of wind</td>
</tr>
<tr>
<td>$\alpha$, $\beta$</td>
<td>parameters of Beta distribution of solar irradiance</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>accelerated factor</td>
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<tr>
<td>$\eta$</td>
<td>penalty parameter</td>
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1 Introduction

The development and utilisation of renewable energy sources (RES) have an impact on modern distribution system in many aspects, e.g. the power flow, protective devices and voltage problems at customers and utility equipment. In these problems, due to the random fluctuant output power of RES, the voltage profile management becomes serious when the penetration of RES is large. Thus, the reactive power optimisation, which is used to compensate the reactive power of the distribution system to manage the voltage profile and reduce the loss, plays an important role [1]. In order to provide reactive power compensation for power losses optimisation, optimal power flow model is generally used to decide the amount of reactive power injected into the distribution grid. Involving the non-linear power flow equations and the random renewable power, it is indeed a stochastic non-linear optimisation (SNO) problem that cannot be solved efficiently. For this reason, much attention has been devoted to developing effective methods for SNO, such as the methods involved approximation treatments to simplify the computation procedure [2, 3].

The power of RES is inherently uncertain, it is not to be sneezed at when more and more RES are integrated into the grid. To handle this problem, a linear stochastic capacitor optimisation model with probabilistic models of RES and linearised DC power flow equations is considered in [4]. In a recent study [5], the researchers used chance constrained programming to solve the reactive power optimisation problem for minimising active power losses.

Moreover, future distribution system with high integration of RES has lots of parameters and variables to be decided, so it is difficult to effectively solve the reactive optimisation problem in a centralised manner. Thus, several techniques on the decentralised optimisation have been proposed in the literature [6, 7]. Since the power flow equations are non-linear, the convergence of the decentralised algorithm can only be guaranteed under some solvability conditions. Different convex relaxation methods [8, 9] have been applied and the distributed optimisation algorithm has been specialised for the resulting convex problem.
The Dantzig–Wolfe decomposition method has been proposed in [10] for decoupling the multi-area reactive power optimisation problem. The author in [7] has developed a decentralised optimal power flow algorithm which is based upon the alternating direction multiplier method (ADMM). As a form of augmented Lagrangian algorithm, the ADMM can solve the convex optimisation problems by breaking them into smaller pieces. Even though in applications where parallel computation of the ADMM is available, the sequential execution of the ADMM counterparts requires some form of global coordination to maintain the pre-defined ordering for the local minimisations and perform the dual updates. More recently, some researchers proposed fully distributed economic dispatch algorithm [11, 12]. Moreover, in [13] applies varying penalty parameter technique to improve the ADMM algorithm, thus the developed reactive power optimisation algorithm can be solved without any central coordinator.

In this paper, we present a communication-efficient distributed algorithm for the reactive power optimisation problem in distribution system with RES. To minimise the power losses and voltage deviation, the reactive power is optimised in a distributed parallel fashion. In addition to the VAR compensation devices, the reactive power of generators is dispatched since the RES-based power systems are allowed to provide reactive power as an ancillary service. First, the reactive power flow equations can be build according to the Ohm’s law and power balance at each bus. Let \( V_j^2 \) be the net injected power is set to be the load minus generation, \( p_j = p_j^0 - p_j^t(q_j = q_j^0 - q_j^t) \).

For every branch in the distribution system, the branch model of power flow equations can be build according to the Ohm’s law and power balance at each bus. Let \( V_j^2 \) be the net injected power is set to be the load minus generation, \( p_j = p_j^0 - p_j^t(q_j = q_j^0 - q_j^t) \).

The generated active (reactive) power on bus \( n_j \), \( p_j^0(q_j^0) \) be the consumed active (reactive) power on bus \( j \). The net injected power is set to be the load minus generation, \( p_j = p_j^0 - p_j^t(q_j = q_j^0 - q_j^t) \).

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2 Preliminary

2.1 Representation of branch power flow based on graph

A distribution system generally has radial topology, which can be described by a directed tree graph \( G = (E, \Psi) \). \( E := \{ n_0, n_1, \ldots, n_n \} \) denotes the vertex set representing the nodes(buses), \( \Psi := \{ \phi_1, \phi_2, \ldots, \phi_n \} \) is the edge set. \( \phi_i = \{(n_{a(i)}, n_{b(i)}) | n_{a(i), b(i)} \in E, n_a(i) \in E \setminus \{n_0\}\} \) denotes the line circuit from bus \( n_{a(i)} \) to bus \( n_b \) where the index \( i \) is defined as the parent node which is closer to the root node \( n_0 \). The root node \( n_0 \) denotes the substation bus in the distribution system which has fixed voltage \( V_0 \).

For the \( j \)th branch (see Fig. 1) in the distribution system, let \( I_j \) be the complex current from buses \( n_{a(j)} \) to \( n_b \), \( S_j = P_j + jQ_j \) be the sending-end complex power from buses \( n_{a(j)} \) to \( n_b \), and \( R(X) \) be the resistance (reactance) on the line from buses \( n_{a(j)} \) to \( n_b \). For each bus, let \( V_j \) be the complex voltage on bus \( n_j \), \( p_j^0(q_j^0) \) be

\[
\begin{align*}
I_j + w_j &= v_{n(j)}, \quad \forall \phi_j \in \Psi \quad (1) \\
P_j - P_{j^+} - R_j(w_j - l_j) &= p_j^0 - p_j^t, \quad \forall \phi_j \in \Psi \quad (2) \\
Q_j - Q_{j^+} - X_j(w_j - l_j) &= q_j^0 - q_j^t, \quad \forall \phi_j \in \Psi \quad (3)
\end{align*}
\]

\( w_j + l_j - v_j = 2(R_j P_j + Q_j X_j) + (R_j^2 + X_j^2)(w_j - l_j) = 0, \quad \forall \phi_j \in \Psi \quad (4)\]

\( (P_j)^2 + (Q_j)^2 + (l_j)^2 = (w_j)^2, \quad \forall \phi_j \in \Psi. \quad (5)\]

Where \( P_j = \sum_{k(n_j, n_k) \in \Psi} P_k \) and \( Q_j = \sum_{k(n_j, n_k) \in \Psi} Q_k \).

2.2 Probability model of injected renewable power

2.2.1 Probability of wind power: The wind speed is described as a random variable, its probabilistic law at a certain location is generally represented by a Weibull distribution, of which the probability density function (PDF) is given by

\[
f_w(v_w) = \left( \frac{k_w}{c_w} \right) \frac{v_w}{c_w}^{k_w - 1} \exp \left( - \frac{v_w}{c_w} \right),
\]

where \( v_w \) is the wind speed (m/s), \( c_w \) and \( k_w \) are the scale factor and the shape factor of Weibull distribution, respectively.

The total active power of the wind turbine can be approximately modelled by a quadratic function of wind speed \( v_w \). Using the quadratic function and (6), the PDF of wind power is estimated analytically [14] as follows

\[
f_p(p) = \begin{cases} 
exp(\frac{p}{c} - \frac{p}{c} - \frac{p}{c}) & \text{if } p = 0 \\
\frac{k_w}{c_w} \left( \frac{p + y}{c} \right)^{k - 1} \exp \left( - \frac{p + y}{c} \right) & \text{if } p \in (0, p_c) \\
de_2 \frac{k_w}{c_w} \left( \frac{p}{c} \right)^{k - 1} \exp \left( - \frac{p}{c} \right) & \text{if } p = p_c,
\end{cases}
\]

where \( p_c \) is the rated power of wind turbine; \( v_{cut} \) is the cut-in wind speed; \( v_{cut} \) is the rated wind speed; \( v_{out} \) is the cut-out wind speed; \( c, k, y \) are

\[
c = \frac{p_c}{v_{cut}^2}, \quad k = \frac{k_w}{2}, \quad y = \frac{-p_c}{v_{cut}^2}
\]

and the parameters \( e_1 \) and \( e_2 \) are

\[
e_1 = 1 - \exp \left( - \frac{v_{cut}}{c_w} \right)^k \quad \text{and} \quad e_2 = \exp \left( - \frac{v_{out}}{c_w} \right)^k
\]

The Dirac’s delta function on the real line which is zero everywhere except at the origin, where it is infinite. Also it is constrained to satisfy the identity \( \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \).
2.2.2 Probability of solar power: The solar irradiance in each time frame is assumed to satisfy the beta distribution [15]

\[ f_p(G) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left( \frac{G}{G_{\text{max}}} \right)^{\alpha-1} \left( 1 - \frac{G}{G_{\text{max}}} \right)^{\beta-1} \]

where \( G \) and \( G_{\text{max}} \) are the actual and maximum solar irradiances, respectively; \( \alpha \) and \( \beta \) are the parameters of the distribution; \( \gamma() \) is the Gamma function.

The PV array is usually composed of L solar modules, each with an area \( A_t \) and efficiency \( n_t \), \( t = 1, \ldots, L \). The PDF of the total power generated by the PV array can be obtained as follows [15]

\[ f_p(p) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left( \frac{p}{p_{\text{max}}} \right)^{\alpha-1} \left( 1 - \frac{p}{p_{\text{max}}} \right)^{\beta-1} \]

where \( p_{\text{max}} = \eta \sum_{i=1}^{L} A_t G_{\text{max}} \) is the maximum power generated by the PV array, \( \eta = (1/\Delta) \sum_{t=1}^{L} A_t \) is the total efficiency of PV arrays.

2.2.3 Probability of RES' power: There are more than one wind turbines and PV arrays connected at a bus in the distribution system. Thus the total injected power of RES at one bus is

\[ p^R_j = p_{w_j} + p_{s_j} \]

where \( p_{w_j} \) is the wind power injected at bus \( n_t \) and \( p_{s_j} \) is the solar power injected at bus \( n_t \). For describing the uncertainties at the renewable power, the PDF of \( p^R_j \) is given by

\[ f_{p^R_j}(p) = f_{p_{w_j}}(p) * f_{p_{s_j}}(p) \]

where the symbol * denotes the convolution integral. Here, the correlation between the random wind power and solar power is ignored. The PDF of the sum of two independent random variables \( p_{w_j} \) and \( p_{s_j} \), each of which has a PDF, is the convolution of their separate density function. It follows from (10) that the CDF of the renewable power can be obtained by the integration

\[ F_{p^R_j}(p') = \text{Pr}(p \geq p') = \int_{p'}^{\infty} f_{p^R_j}(p) dp \]

Assume that the bus \( j \) is provided with WTs and PV arrays. The above, the injected active power \( p^R_j \) randomly varies in the closed interval \([0, \tilde{p}^R_j] \), where \( \tilde{p}^R_j = p_{\text{w_j}} + p_{s_j} \). \( p_{\text{w_j}} \) and \( p_{s_j} \) are the maximum powers of WTs and PV arrays at the \( j \)th bus, respectively.

3 Problem formulation

3.1 Stochastic reactive power optimisation problem

The RES are installed in the distribution system dispersedly, and the fluctuating power generated by these RES brings unfavourable effects on the system voltage profile. Since the penetration of RES can do inverter-based reactive power control with the aid of their separate density function.

Corresponding to the branch power flow model (1)-(4), the variables \((p_j, q_j, w_j, l_j, v_j)\) can be specified for a given \((p_j, q_j)\). Here each bus’s active and reactive powers \((p_j^C, q_j^C)\) are assumed to be known, and the stochastic characteristic of uncertain active generated power \( p_j^G \) is assumed to be known. The design variable is reactive power compensation \( q_j^C \). Let \( S_j := (p_j^C, q_j^C, w_j, l_j, v_j) \) be defined as the state variables of the \( j \)th branch in system. Consider the problem of minimising power loss over the system, the optimisation variables are \( q_j^C := (q_j^C, j \in \Psi) \) and \( S_j := (S_j, j \in \Psi) \) as well. Mathematically, the reactive power optimisation is described as the following SNO problem by considering the uncertain power of RES

\[ \text{SNO:} \quad \min_{S_j \in \Psi} E_{p^G \in \Theta} [f(S_j)] \]

s.t. \( f_1(S_j, p^G_j, q_j^C) = 0; \)

\[ f_2(S_j, p^G_j, q_j^C) = 0; \]

\[ g(S_j, p^G_j, q_j^C) \leq 0. \]

where \( E_{p^G \in \Theta} \) is the expected value operator and \( \Theta \) is the space of uncertain power \( p^G_j \); the objective \( f(S_j) = \sum_{i \in \Psi} (R_i(w_i - l_i) - \gamma_i(v_i - \bar{v})) \) is the active power loss of the line and voltage deviation, \( \bar{v} \) is the reference value for the voltage and \( \gamma_i \) is the weighting parameter which can be modified with respect to the desired importance of the term.

In above SNO problem, the equality constraints (12) denote the power flow (1)-(4), which is linear combination of variables. The equality constraints (13) describe the quadratic non-linear (5). Moreover, the inequality constraints (14) denote the static operation requirements and the limitation of the reactive power penetration

\[ |V_{\text{min}}|^2 \leq v_j \leq |V_{\text{max}}|^2, \quad \forall \psi_j \in \Psi \]

\[ |I_{\text{min}}|^2 \leq w_j - l_j \leq |I_{\text{max}}|^2, \quad \forall \psi_j \in \Psi \]

\[ q_{\text{min}} \leq q_j^C \leq q_{\text{max}}, \quad \forall \psi_j \in \Psi \]

where \( V_{\text{min}} \) and \( V_{\text{max}} \) are the lower and upper bounds of the bus voltage, respectively, \( I_{\text{min}} \) and \( I_{\text{max}} \) are the lower and upper bounds of the branch current, respectively. \( q_{\text{min}} \) and \( q_{\text{max}} \) are the lower and upper bounds of reactive power at bus \( n_j \), respectively.

Due to the non-linear constraints (13) and uncertain renewable power \( p^G_j \), the problem SNO belongs to the SNO problem.

3.2 Multi-scenario formulation of the problem

If the uncertain \( p^G_j \) are replaced with some discretely deterministic states, the constraints with uncertain information can be changed to deterministic constraints with the discrete states, and the expected value operator in the objective function can be calculated by the probability-weighted average of all possible states. Thus, the problem SNO is changed to a deterministic model with multiple different scenarios. To this end, we select scenarios from space \( \Theta \).

3.2.1 Discrete probability model of RES: As stated in the previous section, \( p^G_j \) randomly varies in its closed interval. Moreover, the PDF (10) describes the relative likelihood for the uncertain power \( p^G_j \) to take on a given value in its interval. Before proceeding, it is necessary to discretise the value of \( p^G_j \) into a finite set \( \tilde{p}^G_j := \{a_{j1}, a_{j2}, \ldots, a_{jN}\} \), where \( N \) is the total number of discrete states of the renewable power at the \( j \)th bus. The number of discrete states of renewable power at different bus might be different, it is assumed to be the same in this paper.

The discretisation map \( d[p_{\tilde{p}^G_j}] \) is defined by

\[ d = \begin{bmatrix} a_{j1} & p^G_j = a_{j1}; \ a_{j2} & p^G_j = a_{j2}; \ a_{j3} & p^G_j = a_{j3}; & \ldots; \ a_{jN} & p^G_j = a_{jN} \end{bmatrix} \]

where \( 0 = a_{j1} \leq a_{j2} \leq a_{j3} \leq \ldots \leq a_{jN} = p^G_j \). The \( N \) real scalars divide the region \([0, p^G_j]\) into \( N-1 \) parts. These parts might have non-identical size, such as \( \{a_{j1}, a_{j2}, \ldots, a_{jN}\} = \{0, 0.2, 0.8, 2, 3\} \). In this paper, they are assumed to satisfy \( \delta = a_{j2} - a_{j1} = (p^G_j - 0)/(N - 1) \), for all \( i = 2, \ldots, N \).
In brief, two countable sets \( Y^G_j \) and \( Y^D_j \) present the discrete probability model of RES at the \( j \)th bus, where the former gives the discrete states and the latter gives the probability for discrete state occurrence.

### 3.2.2 Multi-scenario formulation:
Suppose that there are \( M \) buses equipped with RES-based DGs in the distribution system, and \( \{ n_1, n_2, \ldots, n_M \} \) is the set of these buses. The uncertainty in the power generated by these RES-based DGs is represented by the set of scenarios \( \Omega := \{ 1, 2, \ldots, T_m \} \), with associated probabilities \( \{ p_1, p_2, \ldots, p_{T_m} \} \). Each scenario \( \theta \) satisfies (23), (25) and (27).

For each scenario \( \theta \), the probability \( p^G(\theta) \) is

\[
p^G(\theta) = \begin{cases} 
F_G(a_{j,1}) & \text{if } i = 1; \\
F_G(a_{j,i}) - F_G(a_{j,i-1}) & \text{if } i = 2, \ldots, N.
\end{cases}
\]

where\( f_j = (a_{j,1}, \ldots, a_{M,j}) \) (20)

Moreover, the corresponding probability of the scenario is

\[
\rho_\theta = \prod_{j=1}^M \rho_{j,\theta} = \rho_{1,\theta} \cdot \rho_{2,\theta} \cdot \cdots \cdot \rho_{M,\theta}
\]

where \( \rho_{j,\theta} \in Y^D_j \) denotes the probability of occurrence of \( a_{j,\theta} \) [see (19)].

### 4.1 Decomposition of distribution system
Considering the uncertainty of the renewable power, the MSO model of the optimal reactive power problem is presented in the above section. Moreover, the number of scenarios increases exponentially as the number of the buses which is installed DGs. Consequently, in the case of the modern distribution system with plenty of DGs, a number of scenarios are required to keep a certain accuracy. In keeping with the growing complexity of modern distribution system, the decentralised conic optimisation algorithm will be developed to solve the large-scale problem efficiently in the section.

### 4.4 Decentralised strategy for the reactive power optimisation

Motivating by decentralisation of decision-making in organisations or engineering systems, the decomposition is used in the reactive power optimisation problem. It can be seen from Fig. 2 that the decomposition is happened in junction bus \( n_g \). To separate the network, the dummy buses \( n_{in} \) and \( n_{out} \) are inserted into the system.
The common variables consist of the voltage in junction bus given by $Y_{ab} = (P_a(\theta), Q_a(\theta), v_a(\theta), \theta \in \Omega)$. Thus the problem \( CO \) can be rewritten as

$$\min_{Y_{ab} \in D} f_0(Y_a, Y_{ab}) + f_0(Y_b, Y_{ab})$$

subject to $(Y_a, Y_{ab}, Y_b) \in D$ \hspace{1cm} (28)

According to (22), the objective function $f_0$ of problem \( CO \) is split into two parts, i.e. $f_0 = f_0(Y_a, Y_{ab}) + f_0(Y_b, Y_{ab})$.

To handle this situation, the auxiliary variables

$z_{a(b)}^{(u)} = (P_a(\theta), Q_a(\theta), v_a(\theta), \theta \in \Omega)$

$z_{b(a)}^{(v)} = (P_b(\theta), Q_b(\theta), v_b(\theta), \theta \in \Omega)$

in ‘auxiliary problem principle’ \cite{21} are introduced when the region $a$ is separated from $b$. That is to say, copies for common variable $y_{ab}$ are given to the separate parts, respectively.

For the sending subsystem $G_a$ in Fig. 2, $P_a^b + jQ_a^b$ is regarded as the dummy load power at the dummy node $n_{ab}$ that $P_a^b + jQ_a^b = P_a^b + jQ_a^b = P_a + Q_a$. So the net active and reactive powers at the node $n_{ab}$ are equal to the original net power at node $n_a$ plus the dummy load power, i.e.

$$p_a^b(\theta) = p_a(\theta) + P_a^b(\theta), \ \forall \theta \in \Omega$$

$$q_a^b(\theta) = q_a(\theta) + Q_a^b(\theta), \ \forall \theta \in \Omega$$

(29)

For the receiving subsystem $G_b$ in Fig. 2, the dummy node $n_{ab}$ can be considered as a dummy substation whose voltage $V_{ab}^{(u)} = V_{ab}^{(v)}$. In addition, the voltage of dummy substation satisfies

$$w_a(\theta) + f_a(\theta) = v_a(\theta), \ \forall \theta \in \Omega$$

(30)

After decomposition, $z_{a(b)}^{(u)}$ is introduced into the sub-problem of the sending part $G_a$. Meanwhile, $z_{b(a)}^{(v)}$ is introduced into the sub-problem of the receiving part $G_b$. Therefore, the problem (28) is equivalent to

$$\min_{Y_{ab}^{(u)}, Y_{ab}^{(v)}} f_0(Y_a, z_{a(b)}^{(u)}) + f_0(Y_b, z_{b(a)}^{(v)})$$

subject to $(Y_a, z_{a(b)}^{(u)}) \in D_a$, $(z_{b(a)}^{(v)}, Y_b) \in D_b$ \hspace{1cm} (31)

where the equality constraint is to make sure that the copes are equivalent. Two sub-feasible sets $D_b$ and $D_b$ are with respect to $G_a$ and $G_b$, they are given by

$$D_a = \{(Y_a, z_{a(b)}^{(u)}) | (Y_a, z_{a(b)}^{(u)}) \text{ satisfies (23), (25), (27) and (29)}\}$$

$$D_b = \{(Y_b, z_{b(a)}^{(v)}) | (Y_b, z_{b(a)}^{(v)}) \text{ satisfies (23), (25), (27) and (30)}\}$$

It follows from the auxiliary problem principle that the auxiliary Lagrangian function of problem (31) is

$$L_a(Y_a, z_{a(b)}^{(u)}, z_{b(a)}^{(v)}, \lambda) = f_0(Y_a, z_{a(b)}^{(u)}) + f_0(Y_b, z_{b(a)}^{(v)}) + \lambda^T(z_{a(b)}^{(u)} - z_{a(b)}^{(v)}) + \frac{\eta}{2}(z_{a(b)}^{(u)} - z_{a(b)}^{(v)})^2$$

where $\lambda$ is the dual variable and $\eta > 0$ is the penalty parameter. The Lagrangian function $L_a$ is in the augmented form.

Integrating the decomposability of dual ascent with the convergence of the method of multipliers, the algorithm ADMM consists of the iterations \cite{22}

\begin{align*}
L_a(Y_{a+1}, z_{a(b)}^{(u)+1}, z_{b(a)}^{(v)+1}) & = \arg \min_{(Y_{a+1}, z_{a(b)}^{(u)}) \in D_a} L_a(Y_{a+1}, z_{a(b)}^{(u)}, z_{b(a)}^{(v)}, \lambda) \\
L_b(Y_{b+1}, z_{a(b)}^{(u)+1}, z_{b(a)}^{(v)+1}) & = \arg \min_{(Y_{b+1}, z_{b(a)}^{(v)}) \in D_b} L_a(Y_{a+1}, z_{a(b)}^{(u)+1}, z_{b(a)}^{(v)+1}, \lambda) \\
\lambda & = \lambda + \eta(z_{a(b)}^{(u)+1} - z_{a(b)}^{(v)})
\end{align*}

That’s to say, the two subsystems $a$ and $b$ can update their own optimal variables separately and the minimisation steps are performed in a sequential fashion for every subsystems. Partial information of variables, which is about the junction bus between the two neighbouring subsystems, merely need to be exchanged.

### 4.2 Accelerated decentralised algorithm with local domain

The distribution system may be divided into more than two parts. As an example shown in Fig. 3, there is a distribution system in the left half. The system will be separated into six parts, and the junction buses of different parts are $n_1, n_2, n_3, n_4$ and $n_5$. With the help of the auxiliary variables, every part can be considered as distribution subsystems, such as $G_1, G_2, G_3, G_4$ and $G_5$ shown in Fig. 3.

Supposing that the distribution system $G$ is divided into parts $\{G_k, k = 1, \ldots, n\}$, each part is considered as a communication node in the corresponding communication network is $G = (E, \Phi)$ (see Fig. 3c), where $E = \{e_1, \ldots, e_n\}$ is the set of nodes and $\Phi = \{F_1 \times E\}$ is the set of edges.

Let $z_k = \{z_{a(b)}^{(v)} : (\kappa, \kappa') \in \Phi\}$ be the set of the auxiliary variables in subsystem $G_k$, where vector $z_{a(b)}^{(v)}$ denotes the data (include voltage and power) to be transferred from communication node $e_k$ to $e_k$ via the communication link $(c_k, c_k) \in \Phi$. Let $Y_k$ be the core variable of the separate subsystem $G_k$.

\begin{align*}
Y_a &= (P_a(\theta), Q_a(\theta), w_a(\theta), l_a(\theta), q_a^c(\theta), \theta \in \Omega, \psi \in \Psi_a, \psi \in \Psi_a, \{\psi\}) \\
Y_b &= (P_b(\theta), Q_b(\theta), w_b(\theta), l_b(\theta), q_b^c(\theta), \theta \in \Omega, \psi \in \Psi_b, \psi \in \Psi_b, \{\psi\})
\end{align*}
Thus the problem CO can be rewritten as

\[
\min f_{01}(Y_1, z_1) + \cdots + f_{0n}(Y_n, z_n)
\]
\[
\text{s.t. } (Y_k, z_k) \in D_k, \quad \forall k \in E,
\]
\[
z_k^{(k)} = z_k^{(k-1)}, \quad \forall (c_k, c_k) \in \Phi_c
\]  \hspace{1cm} (32)

where \(D_k\) is the \(k\)th feasible set of subsystem \(G_k\).

The above problem (32) is now separable except for the coupled equality constraints requiring the copies to be equal. It should be noted that all the feasible sets \(\{D_k, k = 1, \ldots, n\}\) are convex.

Getting some illumination from the ADMM, a new accelerated augmented Lagrangian decomposition method is derived to tackle the reactive power optimisation problem CO. Therefore, the DCO algorithm is proposed based on the accelerated local Augmented Lagrangian [23]. For \(k = 1, \ldots, n\), the local augmented Lagrangian function with the problem (32) is derived to (33).

\[
\Lambda_k^t(Y_k, z_k, \lambda^t, \lambda^t) = f_{0k}(Y_k, z_k) + \sum_{h \in E_k} (\lambda^t_{k,h})^T z_k^{(k,h)} - \frac{\eta}{2} \|z_k^{(k,h)} - z_k^{(k,h-1)}\|^2_2
\]

where \(\sigma\) denotes the accelerated factor; \(E_k\) is the set of sending subsystems of the \(k\)th subsystem; \(E_k\) is the set of receiving subsystems of the \(k\)th subsystem; and \(E_k = E_k \cup E_k\) is the set of neighbours of the \(k\)th subsystem. The \(\lambda := \{\lambda_{k,h}\}_{h \in E_k} \in \Phi_k\) is the set of the dual variables.

The DCO algorithm consists of iterating on \(t\):

(S-1): Initially set \(t = 0\) and initial value of \(z_k^{(k,0)}\) and \(\lambda^0_{k,h}\) for every \((c_k, c_k) \in \Phi_c\).

(S-2): For every subsystem \(c_k \in E_k\), update \((Y_k^{t+1}, z_k^{t+1})\). First, determine

\[
(Y_k^{t+1}, z_k^{t+1}) := \arg \min_{Y_k, z_k \in D_k} \Lambda_k^t(Y_k, z_k, \lambda^t, \lambda^t)
\]  \hspace{1cm} (34)

then set

\[
z_k^{t+1} := z_k^t + \sigma(z_k^{t+1} - z_k^t)
\]  \hspace{1cm} (35)

(S-3): Update \(\lambda^{t+1}\) according to the auxiliary information of all subsystems \(z^{t+1}\). For every \((c_k, c_k) \in \Phi_c\) set

\[
\lambda_{k,h}^{t+1} := \lambda_{k,h}^t + \eta \sigma \sum_{i \in E_k} s_i(k-h)(z_{k}^{t+1} - z_{k}^{t})
\]  \hspace{1cm} (36)

(S-4): Compute the residual errors

\[
\Delta r_{1} := \max_{(c_k, c_k) \in \Phi_c} \|z_{k}^{t+1} - z_{k}^{t} \|_w
\]

\[
\Delta r_{2} := \max_{(c_k, c_k) \in \Phi_c} \|z_{k}^{t+1} - z_{k}^{t} \|_w
\]

(S-5): \(t = t + 1\), and go to step (S-2) until \(\Delta r_1\) and \(\Delta r_2\) are small enough.

In the proposed DCO algorithm, the initialisation of the auxiliary variables \(z_{k}^{(k,0)}\) includes: the auxiliary voltages are equal to the root bus voltage \(v_0\), the auxiliary powers are equal to zero. It should be noted that the dual variables satisfy \(\lambda_{k,h}(0) = \lambda_{k,h}\) for every \((c_k, c_k) \in \Phi_c\).

In step (S-2), every subsystem finds the solution to the minimisation problem of local augmented Lagrangian function. After the local calculation (34) has been performed, the core variables \(Y_k^{t+1}\) are updated. Moreover, the auxiliary variables \(z_k^{t+1}\) are updated in calculation (35). The dual variables are updated by (36) in step (S-3), where \(s_i(k)\) is the sign function, defined as

\[
s_i(x) = 1 \text{ if } x > 0, \quad s_i(x) = -1 \text{ if } x < 0.
\]

Finally, the residual errors \(\Delta r_1\) and \(\Delta r_2\) are calculated in step (S-4). If the constraints \(z_{k}^{t+1} = z_k^{t+1}\), \(\forall (c_k, c_k) \in \Phi_c\) are satisfied and \(z_k^{t+1}\), then stop (optimal solution found). Otherwise, increase \(t\) by one and repeat the process from step (S-2) to step (S-4).

The DCO algorithm has two parameters: a positive penalty parameter \(\eta\) and a stepsize parameter \(\sigma \in (0, 1)\). Each iteration is comprised of three steps: (i) a minimisation step of the local augmented Lagrangians for every subsystems, (ii) an update step for the primal variables, and (iii) an update step for the dual variables. The computations at each step are performed in a parallel fashion. In addition, the proposed DCO algorithm is convergent since the all the feasible sets \(\{D_k, k = 1, \ldots, n\}\) are convex and the communication network \(G_c\) is connected [24].

Remark 2: The main difference between the algorithm ADMM and DCO is that DCO uses different local augmented Lagrangians in (31)
and the primal update step (35) is local to each subsystem and does not require any message exchanges to be performed. The nodes work sequentially in ADMM algorithm, and the sequential execution requires some form of global coordination to maintain the pre-defined ordering for the local minimisation. This hinders the distributed applicability of such approaches in practical settings. However, the minimisation steps of the DCO algorithm are executed in parallel. Using the DCO algorithm to solve optimisation problems over the corresponding communication network, every node of the network is a processing unit that can only access its own local information as well as information that is available from its neighbours.

5 Case study

5.1 Case study systems and simulation settings

In this study, to assess the performance of the proposed DCO algorithm in solving the optimal reactive power compensation problems, we use the 14-bus distribution system and 69-bus distribution system [25] as the test systems.

For the 14-bus distribution system, as shown in Fig. 4, there were five buses installed renewable energy based DGs, i.e. 3, 2, 5, 4, 2.1 MW DGs were located at buses $n_3$, $n_5$, $n_4$, $n_2$ and $n_{13}$, respectively, and two buses installed VAR compensators, i.e. 3 and 6 Mvar compensators were connected at buses $n_5$ and $n_7$, respectively. For the 69-bus distribution system, as shown in Fig. 5, there were 14 buses installed DGs and VAR compensators, i.e. 100, 100, 400, 1200 and 200 kW DGs were located at buses $n_{10}$, $n_{20}$, $n_{40}$, $n_{60}$ and $n_{63}$, respectively; 200, 500 and 100 kvar compensators were located at buses $n_{11}$, $n_{33}$, $n_{53}$, $n_{64}$, respectively.

The splitting of the distribution system is illustrated in Table 1, where the 14-bus system is split into six subsystems and the edge set of each subsystem is listed in the second column in the table, the 69-bus system is split into seven subsystems and the edge set of each subsystem is listed in the fourth column in the table.

The specifications of the RES-based DG are as follows: for the 500 kW wind turbine, the cut in wind speed is 3 m/s, the rated wind speed is 10.5 m/s and the cut off wind speed is 30 m/s; the efficiency of the PV module is 15.8%. According to the statistical analysis, the parameters of the distribution of the wind power and solar power are $c = 7.0332$, $k = 2.6194$ and $\alpha = 4.9828$, $\beta = 2.3417$.

5.2 Test results

5.2.1 Multi-scenario model of uncertainty: In terms of (7) and (9) which discussed in Section 2, the PDF of wind power and solar power can be given respectively. Considering the uncorrelated wind power and solar power, the distribution of the sum of the two is obtained by the convolution in (10). As can be seen in Fig. 6, the dash curve represents the PDF of the output power of a WT and the solid curve represents the PDF of the output power of PV arrays.

The number of discrete states is set to be 11 ($N = 11$), which means that $\delta = 0.1$ p.u. and the set of discrete states is $I_N = \{0, 0.1 \text{ p.u.}, \ldots, 0.9 \text{ p.u.}, 1 \text{ p.u.}\}$. Moreover, according to (18) and (19), the multi-scenario model of RES-based DG is shown in Table 2, where the discrete states are listed in the second

### Table 1 Splitting of 14-bus and 69-bus distribution systems

<table>
<thead>
<tr>
<th>Subsystem (14-bus)</th>
<th>Edge set $\Psi_i$</th>
<th>Subsystem (69-bus)</th>
<th>Edge set $\Psi_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>${\psi_1, \psi_2, \psi_3}$</td>
<td>1</td>
<td>${\psi_1, \psi_2, \psi_3, \psi_4}$</td>
</tr>
<tr>
<td>2</td>
<td>${\psi_4, \psi_5}$</td>
<td>2</td>
<td>${\psi_5, \psi_6}$</td>
</tr>
<tr>
<td>3</td>
<td>${\psi_7, \psi_8}$</td>
<td>3</td>
<td>${\psi_9, \psi_{10}}$</td>
</tr>
<tr>
<td>4</td>
<td>${\psi_{11}, \psi_{12}, \psi_{13}}$</td>
<td>4</td>
<td>${\psi_{14}, \psi_{15}}$</td>
</tr>
<tr>
<td>5</td>
<td>${\psi_{16}}$</td>
<td>5</td>
<td>${\psi_{17}, \psi_{18}}$</td>
</tr>
<tr>
<td>6</td>
<td>${\psi_{19}, \psi_{20}}$</td>
<td>6</td>
<td>${\psi_{21}, \psi_{22}, \psi_{23}}$</td>
</tr>
<tr>
<td>7</td>
<td>${\psi_{24}, \psi_{25}}$</td>
<td>7</td>
<td>${\psi_{26}, \psi_{27}, \psi_{28}}$</td>
</tr>
</tbody>
</table>

### Table 2 The multi-scenario model of RES-based DG

<table>
<thead>
<tr>
<th>Index $i$</th>
<th>Discrete states, $p.u.$</th>
<th>Probability of wind power</th>
<th>Probability of solar power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.1018</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.1489</td>
<td>0.0002</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.1441</td>
<td>0.0025</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.1278</td>
<td>0.0137</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.1077</td>
<td>0.0417</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>0.0878</td>
<td>0.0894</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>0.0696</td>
<td>0.1507</td>
</tr>
<tr>
<td>8</td>
<td>0.7</td>
<td>0.0640</td>
<td>0.2079</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
<td>0.0411</td>
<td>0.2328</td>
</tr>
<tr>
<td>10</td>
<td>0.9</td>
<td>0.0308</td>
<td>0.1915</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0.0864</td>
<td>0.0696</td>
</tr>
</tbody>
</table>
The corresponding occurrence probability of wind power and solar power are listed, respectively, in the third and fourth columns of the table.

The number of the scenarios increases exponentially as the number of the buses which is installed DGs. For example, there are 11⁵ scenarios if five buses are installed DGs in the 14-bus distribution system. However, by the time the increasing number of RES-based DGs is connected into the distribution system, a large-scale optimisation problem of power flow is inevitable. Thus the presented DCO algorithm is applied to solve the MSO model of reactive power optimisation in distribution system with DGs.

5.2.2 Results regarding the voltage after optimisation: After the reactive power optimisation in the distribution system considering the uncertainty of renewable energy, Fig. 7 illustrates the bus voltage fluctuation of 14-bus and 69-bus distribution systems, respectively, and Fig. 8 illustrates the bus voltage profile in the worst-case scenario, where the solid line denotes the bus voltage before the reactive power optimisation and the dash line denotes the bus voltage profile after the optimisation. As you can see from the figures that the voltage has been improved and it fluctuated in a tolerable region.

5.2.3 Comparison between the distributed algorithm and the centralised algorithm: To verify the ability of the proposed DCO algorithm, the centralised algorithm has also been applied to solving the CO problem. Here the interior point method is used to solve the convex CO problem in centralised manner. Moreover, the results of centralised and distributed algorithm are summarised in Table 3. In addition to the improvement of bus voltage, the reactive power dispatching minimises the active power losses of distribution system. It can be seen that the DCO algorithm can achieve the optimal solution of the CO problem. All sub-problems are solved in parallel, the execution time of the distributed solution is lower than the one of the centralised solution. It can be seen that the DCO algorithm has much better performance in comparison to the centralised algorithm since the DCO algorithm can reach an optimal solution in a shorter time.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Power losses, kW</th>
<th>14-bus system</th>
<th>69-bus system</th>
<th>Execution time, s</th>
<th>14-bus system</th>
<th>69-bus system</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCO algorithm</td>
<td>34.23</td>
<td>59.01</td>
<td>0.23</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>centralised algorithm</td>
<td>34.22</td>
<td>59.05</td>
<td>2.08</td>
<td>13.78</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7  Bus voltage fluctuation after optimisation

a 14-bus system  
b 69-bus system

Fig. 8  Voltage profile in the worst case scenario before and after optimisation

a 14-bus system  
b 69-bus system
5.2.4 Convergence: To verify the convergence of the DCO algorithm (S-1)–(S-5), Fig. 9 shows the relative error $\|Y_t - Y^*\|_\infty$ versus the number of iterations while the algorithm is applied for the 14-bus distribution system as shown in Fig. 4. Here, $Y_t$ is the solution of DCO algorithm at the end of the $t$th iteration, and $Y^*$ is the optimal solution of CO problem, computed in a centralised way. It can be seen from Fig. 9 that the proposed DCO algorithm requires several iterations to let any relative error small enough.

In addition, Fig. 10 shows the results for distribution system of different sizes. An encouraging observation is that network size does not appear to affect speed of convergence dramatically. Although the increasing number of buses grows the scale of the centralised problem sharply, the distributed sub-problems share the complexity and can obtain the optimal solution efficiently due to its convexity, furthermore the convergence rate of DCO algorithm appears unaffected. The proposed DCO algorithm requires about 40 iterations to achieve any relative error below $10^{-3}$.

The performance of the proposed DCO algorithm and ADMM algorithm was compared in Fig. 11. The two algorithms were applied for the 69-bus distribution system with uncertain RES, where the DCO algorithm was implemented for $\sigma = 0.3$ and the penalty parameter and initialisation points were the same in the two algorithms. It can be noticed that the DCO algorithm performs significantly better than ADMM.

6 Conclusion

In order to reduce the power loss and voltage deviation in distribution system with RES, this paper proposes a communication-efficient distributed algorithm for reactive power optimisation problem based on the accelerated local augmented Lagrangian. To handle the non-linear constraints and uncertainties in the problem, the linear conic model of power flow equations and discrete probability model of RES are applied such that the problem is changed to a multi-scenario CO problem. Besides, in the proposed distributed algorithm, the auxiliary problem principle and augmented Lagrangian decomposition have been utilised to separate the large-scale problem into some smaller decomposed sub-problems, and the optimal solution can be obtained by the coordination of sub-problems in the absence of a central processing unit. Compared with the algorithm in central manner, the decentralised algorithm greatly improves the computation efficiency. Moreover, it is desirable to improve the communication efficiency, because subsystems only talk to their neighbours for the information and the exchanged information is the parts that are related to their neighbours. The proposed algorithm is tested on the distribution systems of different sizes with uncertain wind and PV power. The simulation results demonstrate the effectiveness of the proposed algorithm. Basing on the research results of this paper, further research will be done on the economic dispatching or demand response problem that manage all controllable devices under the uncertainties of the renewable power and the load.

7 Acknowledgment

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8 References

Appendix

Given the distribution network $G$, the impedances $\{Z_j\}_{j \in \Psi}$ ($Z_j = R_j + jX_j$) and the feeder voltage $V_0$, then the other variables including the power flows, the voltages, the currents, and the bus net loads satisfy the following physical laws for all branches. Here some variables are complex variables, such as $s = p + jq$, and $S = P + jQ$.

Definition of the branch power flow

$$S_j = V_0^* P_j^*$$

(37)

Ohm’s law

$$V_{n(j)} - V_j = Z_j I_j$$

(38)

Power balance

$$S_j - Z_j |I_j|^2 - S_{k=1} = s_j$$

(39)

where $S_{k=1} = \sum_{(k,n,j) \in \Psi} S_k$.

Substituting (37) into (38) yields

$$V_{n(j)} - V_j = Z_j \frac{S_j^*}{V_0}$$

(40)

Taking the magnitude squared of (37) and (40), we have

$$|S_j|^2 = |V_{n(j)} I_j^*|^2$$

(41)

$$|V_{n(j)}|^2 - |V_j|^2 + |Z_j|^2 |I_j|^2 - (Z_j S_j^* + Z_j^* S_j) = 0$$

(42)

In terms of real variables and set $V_j := |V_j|^2$, $w_j := 0.5 \times (|V_{n(j)}|^2 + |I_j|^2)$ and $l_j := 0.5 \times (|V_{n(j)}|^2 - |I_j|^2)$, we therefore have

$$l_j + w_j = V_0 \forall \psi \in \Psi$$

(43)

$$P_j = P_{j=} - R_j (w_j - l_j) = P_j^0 - P_j^G \forall \psi \in \Psi$$

(44)

$$Q_j - Q_{j=} - X_j (w_j - l_j) = Q_j^0 - Q_j^G \forall \psi \in \Psi$$

(45)

$$w_j + l_j - v_j - 2( R_j P_j^0 + X_j Q_j^0) + (R_j^2 + X_j^2)(w_j - l_j) = 0 \forall \psi \in \Psi$$

(46)

$$(P_j)^2 + (Q_j)^2 + (l_j)^2 = (w_j)^2 \forall \psi \in \Psi$$

(47)

9 Appendix

Given the distribution network $G$, the impedances $\{Z_j\}_{j \in \Psi}$ ($Z_j = R_j + jX_j$) and the feeder voltage $V_0$, then the other variables including the power flows, the voltages, the currents, and the bus net loads satisfy the following physical laws for all branches. Here some variables are complex variables, such as $s = p + jq$, and $S = P + jQ$.

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where $S_{k=1} = \sum_{(k,n,j) \in \Psi} S_k$.

Substituting (37) into (38) yields

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(40)

Taking the magnitude squared of (37) and (40), we have

$$|S_j|^2 = |V_{n(j)} I_j^*|^2$$

(41)

$$|V_{n(j)}|^2 - |V_j|^2 + |Z_j|^2 |I_j|^2 - (Z_j S_j^* + Z_j^* S_j) = 0$$

(42)

In terms of real variables and set $V_j := |V_j|^2$, $w_j := 0.5 \times (|V_{n(j)}|^2 + |I_j|^2)$ and $l_j := 0.5 \times (|V_{n(j)}|^2 - |I_j|^2)$, we therefore have

$$l_j + w_j = V_0 \forall \psi \in \Psi$$

(43)

$$P_j = P_{j=} - R_j (w_j - l_j) = P_j^0 - P_j^G \forall \psi \in \Psi$$

(44)

$$Q_j - Q_{j=} - X_j (w_j - l_j) = Q_j^0 - Q_j^G \forall \psi \in \Psi$$

(45)

$$w_j + l_j - v_j - 2( R_j P_j^0 + X_j Q_j^0) + (R_j^2 + X_j^2)(w_j - l_j) = 0 \forall \psi \in \Psi$$

(46)

$$(P_j)^2 + (Q_j)^2 + (l_j)^2 = (w_j)^2 \forall \psi \in \Psi$$

(47)