Large-Scale Power System Robust Stability Analysis Based on Value Set Approach

Jinghao Zhou, Peng Shi, Deqiang Gan, Ying Xu, Huanhai Xin, Changming Jiang, Huan Xie, Tao Wu

Abstract—This paper presents a method for robust stability analysis of large-scale power systems based on the value set approach. The order of the system is first reduced by the dimension reduction and modal truncation to capture the oscillation modes of interest. The characteristic polynomial is then obtained by the diagonal expansion, which allows the structure of coefficient functions to be exploited. The Mikhailov plot is generated with ease using the edge theorem and mapping theorem, and by inspection of the plot, the uncertainty’s impact becomes transparent. The performance of the proposed method is tested on a 547-machine 8647-bus model of the actual North China system. The results of several case studies are reported, and related works are reviewed for comparison.

Index Terms—Edge theorem, Mikhailov plot, parametric uncertainty, robustness analysis, small signal stability, value set.

I. INTRODUCTION

The problem of low-frequency oscillations long exists, threatening the stability of large-scale power systems. Changes in operating conditions and network configuration adds more complications to the oscillation, which calls for the understanding and assessment of the uncertainties. By analyzing the impact of uncertainty, measures can be taken accordingly to improve the damping, while insights are gained into power system operations and planning.

Various methods have found use in the uncertainty analysis of power system low-frequency oscillations. The structured singular value (SSV) theory proved useful in evaluating the robust stability of large-scale power systems [1-4]. The Kharitonov theorem was applied in the robust design of controller parameters, where the uncertainties were represented by an interval polynomial [5-9]. In [10], the stability radius theory was used to obtain the robust small-signal stability region of the wind generation uncertainty. Yet relatively little work has unraveled the impact of each parameter on system stability in large-scale power systems.

In this paper, we probe into the impact of uncertainty on large-scale power system stability using the value set approach [11]. The parametric uncertainty in operating conditions is first modelled in a state-space representation and then “pulled out” into the standard M-Δ framework. The dimension of the M-Δ framework and the order of its transfer function are reduced to capture the oscillation modes of interest. Using the diagonal expansion formula for matrix determinants, the characteristic polynomial is then computed efficiently. Finally, with the celebrated edge theorem and mapping theorem, the Mikhailov plot (i.e., the value sets of the characteristic polynomial) is drawn to facilitate robustness analysis. The value set approach permits one to analyze the respective impact of each system operating parameter (i.e., uncertainty) on small-signal stability. The results of the actual 547-machine 8647-bus North China system are reported to demonstrate the practicality of the suggested approach.

II. ROBUSTNESS ANALYSIS BY VALUE SETS

The value set approach [11] is a graphics-oriented method for robustness analysis based on the characteristic polynomial of the system. The main ideas include the Mikhailov plot—an easy-to-interpret figure that makes uncertainty’s impact transparent—along with the edge theorem and mapping theorem—two useful tools that map parameter ranges to polygons in the Mikhailov plot.

We first write the characteristic polynomial of the uncertain system as

$$P(s, \delta) = a_n(\delta) + a_{n-1}(\delta) \cdot s + \cdots + a_1(\delta) \cdot s^n,$$

where $\delta = [\delta_1, \delta_2, \ldots, \delta_m]^T$ is a vector of uncertain parameters, and $a_i(\delta), i = 1 \ldots n$ are the coefficient functions subject to $\delta$.

Suppose the uncertain parameter vector $\delta$ is bounded by an operating domain $D$, which is a hyper rectangle

$$D = \{\delta | \delta_i \in [\delta_{i-}, \delta_{i+}], i = 1 \ldots m\}.$$

Then the stability of the polynomial family $P(s, D) = \{P(s, \delta) | \delta \in D\}$ can be determined by a simple criterion as follows.

**Theorem 1 (boundary crossing theorem):** The family of polynomials $P(s, D) = \{P(s, \delta) = \sum a_i(\delta) s^i | \delta \in D\}$ with continuous real coefficient functions $a_i(\delta)$ is robustly stable, if and only if: (i) There exists a stable polynomial $p(s, \delta) \in P(s, D)$, (ii) $j \omega \not\in \text{Roots}[P(s, D)]$ for all $\omega \geq 0$. (Roots[$P(s, D)$] denotes the set of all roots of $P(s, \delta)$ for all $\delta \in D$.)

The above theorem is visually explanatory: the roots of $P(s, \delta)$ cannot jump from the left half plane to the right half plane without crossing the $j\omega$-axis. Therefore, if $P(s, D)$ is found to

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J. Zhou, P. Shi, D. Gan, and H. Xin are with School of Electrical Engineering, Zhejiang University, Hangzhou, China (zhoujinghao@zju.edu.cn, shipeng@zju.edu.cn, deqiang.gan@ieee.org, xinhb@zju.edu.cn)

Y. Xu, C. Jiang are with North China Branch, State Grid Corporation of China, Beijing, China. (xu.yang@nc.sgcc.com.cn, jiang.changming@nc.sgcc.com.cn)

H. Xie, T. Wu are with North China Electric Power Research Institute Co., Ltd., Beijing, China. (xie.huan@nc.sgcc.com.cn, wu.tao@ncepti.com.cn)
contain no root on the \( j\omega \)-axis, then no crossing of the \( j\omega \)-axis occurs, and all the roots of \( P(s,D) \) must reside in the left half plane.

To check whether \( P(s,D) \) contains any root on the \( j\omega \)-axis, however, is not easy. A more practical approach is to evaluate \( P(s,D) \) along the \( j\omega \)-axis and check whether the value set (i.e., image set)

\[
P(j\omega, D) = \{ P(j\omega, \delta) \mid \delta \in D \}
\]

contains the number zero. If the value set \( P(j\omega, D) \) does not contain the zero, then \( P(s,D) \) must have no root at \( s = j\omega \).

**Theorem 2 (zero exclusion):** Given a polynomial family \( P(s,D) = \{ P(s,\delta) \mid \delta \in D \} \), the set \( P(s,D) \) is robustly stable, if and only if:

1. There exists a stable polynomial \( p(s,\delta) \in P(s,D) \).
2. \( 0 \not\in P(j\omega, D) \) for all \( \omega \geq 0 \).

A convenient way to apply the above theorem is to plot the value set \( P(j\omega, D) \) in the complex plane and check whether the images envelop the origin. Instead of plotting the value sets at all \( \omega \geq 0 \), a sufficiently dense grid of \( \omega \) in the frequency range of interest will suffice.

The frequency plot \( P(j\omega, D) \) on a grid of \( \omega \) in the complex plane is referred to as a Mikhailov plot. For a robustly stable polynomial, the Mikhailov plot starts on the positive real axis and encircles the origin in a counterclockwise direction as \( \omega \) increases.

Here we take a 4th degree polynomial for example:

\[
P(s, \delta) = 24s^4 + (60\delta + 72)s^3 + (57-5\delta^2)s^2 + (50\delta + 60)s + 10,
\]

where the uncertain parameters \( \delta_1, \delta_2 \in [-1, 1] \). The Mikhailov plot of \( P(j\omega, D) \) for \( 0.05 < \omega < 1.7 \) is shown in Fig. 1. Since the origin is excluded from the value sets, the polynomial proves robustly stable by theorem 2. (For \( \omega > 1.7 \), the value sets depart farther away from the origin and is not shown in Fig. 1.)

The origin in the Mikhailov plot serves as a critical point for \( P(s,D) \)'s stability: if the plot contains the origin, then some roots of \( P(s,\delta) \) must cross the \( j\omega \)-axis as \( \delta \) varies and \( P(s,D) \) is not robustly stable. The minimal distance between the Mikhailov plot and the origin serves as a relative measure of the polynomial’s stability: the closer the plot is to the origin, the less stable the polynomial tends to be. In particular, if the plot \( p(j\omega) \) passes through the origin, then the polynomial \( p(s) \) has roots on the \( j\omega \)-axis and is on the threshold of instability. Therefore, by inspection of \( P(j\omega, \delta) \)'s movement within the value set as parameters \( \delta \) vary, the impact of uncertainty can be revealed.

Based on the zero exclusion principle (theorem 2), the Mikhailov plot needs to be computed on a dense grid of parameters in the frequency range of interest. As the number of uncertain parameters rises, the computational effort blows up. Fortunately, the value set at each frequency can be constructed with great ease if the polynomial has special structures.

The celebrated edge theorem states that, if the coefficients \( a_i(\delta) \) of a polynomial family \( P(s,D) = \{ P(s,\delta) = \sum a_i(\delta)s^i \mid \delta \in D \} \) are affine functions of uncertain parameters \( \delta \) (i.e., the coefficients \( a_i(\delta) \) depends linearly on the parameters. For example, \( a_2(\delta) = 2\delta_1 + \delta_2 - 1 \), then the value set of the polynomial at each frequency is a convex polygon. With this theorem, one need only compute the images of vertices to construct the Mikhailov plot.

**Theorem 3 (edge theorem):** The value set of the polynomial family \( P(s,D) = \{ P(s,\delta) = \sum a_i(\delta)s^i \mid \delta \in D \} \) with affine coefficient functions \( a_i(\delta) \) for fixed frequency is always a convex polygon whose vertices are generated by the vertices of \( D \). When \( \delta \) varies along any single coordinate of \( D \), the image \( P(s,D) \) varies along a straight line in the complex plane.

In fact, the example polynomial (4) has exactly the affine coefficient functions stated in the above edge theorem, and its value sets shown in Fig. 1 are convex polygons generated by computing the images \( P(s,\delta) \) only at \( \delta_1, \delta_2 = \pm 1 \).

If the coefficient functions are multilinear (i.e., terms like \( \delta_1\delta_2, \delta_3\delta_4, \delta_5\delta_6\delta_7 \)), then a weaker result is available as follows.

**Theorem 4 (mapping theorem):** The convex hull of the value set of the polynomial family \( P(s,D) = \{ P(s,\delta) = \sum a_i(\delta)s^i \mid \delta \in D \} \) with multilinear coefficient functions \( a_i(\delta) \) is the convex hull of the images of the vertices of \( D \).

Theorem 3 and 4 offer a drastic simplification of value set construction when the characteristic polynomial, usually in complicated structures, is approximated with affine or multilinear coefficient functions (see III-D). Theorem 4 provides a useful sufficient stability condition for theorem 2, but becomes inconclusive when the convex hull envelopes the origin. When the envelopment occurs, the actual value set at that frequency should be checked for zero exclusion.

### III. POWER SYSTEM UNCERTAINTY MODELLING AND VALUE SET CALCULATION

In this section, we detail the uncertainty modelling techniques to derive the characteristic polynomial of large-scale power systems before applying the value set approach.

#### A. State-Space Uncertainty Representation

The uncertainties in operating conditions and controls can be reflected analytically in the state-space matrix, whose elements are functions of the uncertain parameters and can be approximated by low order polynomials with good accuracy [1].

Suppose the state-space representation of the uncertain power system is given by
\[ \dot{x} = A_p(p) \cdot x, \]  
(5)

where \( x \) is the state vector, \( A_p(p) \) is the parameter-dependent state matrix, and \( p = [p_1, p_2, \ldots, p_m] \) is a set of \( m \) real uncertain parameters that are assumed to vary within some specific limits \( p_k^{\text{min}} \leq p_k \leq p_k^{\text{max}} \).

To simplify the analysis, the range of uncertain parameters is preferred to be normalized to the interval \([-1,1]\). Express \( p_k \) as

\[
p_k = \frac{p_k^{\text{min}} + p_k^{\text{max}}}{2} + \frac{p_k^{\text{max}} - p_k^{\text{min}}}{2} \delta_k,
\]

(6)

where \( \delta_k \) is a real scalar within the interval \([-1,1]\). Hence, the variation in \( p_k \) is captured by the variation in \( \delta_k \), where \( p_k = p_k^{\text{max}} \) for \( \delta_k = 1 \), and \( p_k = p_k^{\text{min}} \) for \( \delta_k = -1 \).

Thus, the state-space representation of (5) is rewritten as

\[ \dot{x} = A_{\delta}(\delta) \cdot x \]

(7)

Each element \( a_{ij} \) of the uncertain state matrix \( A_{\delta}(\delta) \) can be approximated by a low order polynomial. Take a linear approximation for example (for cases of higher order, see [1]),

\[
a_{ij} = f(\delta) \approx \hat{a}_{ij0} + \hat{a}_{ij1} \delta + \hat{a}_{ij2} \delta^2 + \cdots + \hat{a}_{ijm} \delta^m,
\]

(8)

where \( \hat{a}_{ij0}, \hat{a}_{ij1}, \ldots, \hat{a}_{ijm} \) are unknown constants depending on the operating condition, which are determined by a least-squares solution as follows [1].

1) A parameter space corresponding to different operating conditions is selected as \( \delta_k \in \{\delta_{k1}, \delta_{k2}, \ldots, \delta_{km}\}, \ k = 1 \ldots m, \) where \( \delta_i (1 < i < n) \) is a real number within \([-1,1]\). Each uncertain parameter \( \delta_i \) varies independently and discretely within the parameter space.

2) Power flow calculation is conducted at all possible combinations of the parameter values \( (\delta_{i1}, \delta_{i2}, \ldots, \delta_{im}) \) to obtain the state matrix \( A_{\delta}(\delta) \), where subscript \( (i) \) denotes the \((i)\)-th possible combination, and \( \delta_{i(k)} \) denotes the value of \( \delta_k \) in the \((i)\)-th possible combination.

3) The polynomial coefficients of (8) are obtained by solving the following linear equations

\[
\begin{bmatrix}
1 & \delta_{i1} & \cdots & \hat{a}_{ij0} \\
1 & \delta_{i2} & \cdots & \hat{a}_{ij1} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \delta_{ic} & \cdots & \hat{a}_{ijm}
\end{bmatrix}
= \begin{bmatrix}
a_{ij1} \\
a_{ij2} \\
\vdots \\
a_{ijm}
\end{bmatrix},
\]

(9)

or, abbreviated as

\[
P \hat{a} = y,
\]

(10)

where \( a_{i(j)} \) is the \((i,j)\)-th element of the state matrix \( A_{\delta}(\delta) \), and \( C \) is the number of possible combinations of the parameter values.

The unknown vector \( \hat{a} = [\hat{a}_{i1}, \hat{a}_{i2}, \ldots, \hat{a}_{im}]^T \) is solved by a least-squares solution as follows

\[
\hat{a} = P^\dagger y = (P^T P)^{-1} P^T y,
\]

(11)

where \( P^\dagger \) is the Moore-Penrose pseudo-inverse of the matrix \( P \).

The uncertain system of (7) is now represented as

\[
\dot{x} = A_{\delta}(\delta) \cdot x \approx \sum_{k=1}^{m} \hat{A}_{\delta_k}(\delta_l) x,
\]

(12)

where \( \hat{A}_{\delta_k} = [\hat{a}_{i(k)}] \ (0 \leq k \leq m) \) is a constant matrix with elements \( \hat{a}_{i(k)} \), and \( I \) is an identity matrix.

The above procedure is executed element by element in a for-loop and is sped up by the Parallel Computing Toolbox of MATLAB. The sparsity of the state matrix \( A_{\delta}(\delta) \) can also be exploited, which, along with the parallel computation, allows significant computing time to be saved.

B. Uncertainty Separation and Dimension Reduction

To decipher the characteristic polynomial of large-scale power systems, the uncertain parameters in (12) are preferably pulled out into a separate diagonal matrix with lower dimensions. The transformed system with uncertainty pulled out—as shown in Fig. 2(b)—is called the \( M-\Delta \) framework [1-4], where the transfer function matrix \( M(s) \) is a fixed matrix independent of the uncertain parameters \( \delta \), while the transfer function matrix \( \Delta(s) \) is an uncertain diagonal matrix with \( \delta \) on its main diagonal. The derived \( M-\Delta \) system has the same characteristic polynomial as that of the original system, and it enjoys some structural advantages in the characteristic polynomial calculation (see III-D).

To derive the \( M-\Delta \) system, we first apply the singular value decomposition (SVD) to simplify (12) so that the dimensions of the derived \( M(s) \) and \( \Delta(s) \) are reduced.

Factorize the coefficient matrix \( \hat{A}_i \) of (12) by the SVD as

\[
\hat{A}_i = U \Sigma V^H, \]

(13)

where \( U_i \) is a diagonal matrix with the singular values in descending order, \( U_i, V_i \) are unitary matrices, and the superscript \( H \) stands for the Hermitian transpose.

By ignoring the small singular values in \( \Sigma_i \) and eliminating the corresponding columns of \( U_i \) and \( V_i \), the matrix \( \hat{A}_i \) can be approximated by

\[
\hat{A}_i = [U_{i1} \quad U_{i2}] \begin{bmatrix}
\Sigma_{i1} & 0 \\
0 & \Sigma_{i2}
\end{bmatrix} [V_{i1} \quad V_{i2}]^H, \]

(14)

Given the desired dimensions of \( U_{i1}, \Sigma_{i1}, \) and \( V_{i1}, \) the above representation is shown to be the best approximation of matrix \( \hat{A}_i \) in terms of the root-mean-square error [12].

Now system (12) becomes

\[
\dot{x} = \hat{A}_\delta x + \sum_{k=1}^{m} \hat{A}_{\delta_k}(\delta_l) x,
\]

(15)

and we formulate the \( M-\Delta \) system as follows.

1) Introduce fictitious extra input and output vectors \( z = [V_{i1} \quad V_{i2}]x \) and \( w = \Delta x, \) where \( \Delta \) is a diagonal matrix defined by \( \Delta = \text{diag}(\delta_{i1}, \delta_{i2}, \ldots, \delta_{im}) \). We express system (15) as in Fig. 2(a) [2],

![Diagram](image-url)
\[
\begin{align*}
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} &= \begin{bmatrix}
\Gamma_{11} & \Gamma_{12} \\
\Gamma_{21} & \Gamma_{22}
\end{bmatrix}
\begin{bmatrix}
x \\
w
\end{bmatrix},
\end{align*}
\]  
(16)

\[
\begin{align*}
w &= \Delta z,
\end{align*}
\]  
(17)

where \( \Gamma_{11} = \hat{A}_0 \), \( \Gamma_{12} = [U_1 \Sigma_1 U_2 \Sigma_2 \ldots U_m \Sigma_m] \), \( \Gamma_{21} = [V_1 V_2 \ldots V_m]^{\dagger} \), and \( \Gamma_{22} = 0 \). The matrix equation \( \dot{x} = \Gamma_{1}(x + \Gamma_{2})w \) is identical to (15).

2) The transfer function matrix \( \tilde{M}(s) \) from \( w \) to \( z \) is now computed by taking the Laplace transform of (16):

\[
\tilde{M}(s) = \Gamma_{22} + \Gamma_{21} \frac{1}{s} (I - \frac{1}{s} \Gamma_{11})^{-1} \Gamma_{12}.
\]  
(18)

It becomes clear that the significance of approximation (14) is to reduce the dimensions of the input and output vectors \( z, w \).

The dimension of \( \tilde{M}(s) \), determined by the number of singular values retained in \( \Sigma \) \((k = 1 \ldots m)\), can be remarkably lower than that of \( A \). Nevertheless, the entries of the transfer function matrix \( \tilde{M}(s) \) may still be crammed with high-order transfer functions that defy analysis when system scale is too large; a model order reduction is therefore suggested in the next section.

C. Model Order Reduction

A full-order characteristic polynomial is not necessary nor preferred for the application of the value set approach, because a slight change in the frequency \( \omega \) will result in a significant change in \( P(j \omega, D) \), which not only obscures uncertainty’s impact on the modes of interest but renders the Mikhailov plot difficult to read. Various model order reduction methods are available in the literature [13] to reduce the order of transfer functions. Here we apply the simple modal truncation [14] to capture the oscillation modes of interest.

Rewrite (18) as

\[
\tilde{M}(s) = \Gamma_{22} + \Gamma_{21} (sI - \Gamma_{11})^{-1} \Gamma_{12}.
\]  
(19)

Factorize \( \Gamma_{11} \) by eigen decomposition as

\[
\Gamma_{11} = \Lambda X Y^{T},
\]  
(20)

where \( \Lambda \) is a diagonal matrix containing the eigenvalues of \( \hat{A}_0 \), \( X \) is a square matrix whose columns are the corresponding right eigenvectors, and \( Y^{T} = X^{-1} \).

\( \tilde{M}(s) \) is now expressed as a partial fraction expansion [15]:

\[
\tilde{M}(s) = \Gamma_{22} + \Gamma_{21} X(sI - \Lambda)^{-1} Y^{T} \Gamma_{12}
\]  
(21)

where \( n \) is the number of eigenvalues, \( R_i = \Gamma_{21} X_i Y_i^{T} \Gamma_{12}, \) and \( X_i, Y_i \) denote the \( i \)-th columns of \( X, Y \) respectively.

The terms in (21) with smaller contributions to the transfer function, i.e., with lower values of \( \| R_i \|_2 / \| R_i(\lambda_i) \|_2 \), can therefore be truncated to produce a reduced-order system \( \tilde{M}(s) \) with similar dynamic behavior to that of \( \tilde{M}(s) \) [16].

Let \( \Lambda \) denote a diagonal matrix containing the preserved eigenvalues \( \lambda_1, \ldots, \lambda_k \) from \( \Lambda \), and \( X, Y \) are formed by retaining the corresponding columns of \( X, Y \) respectively. We can write the reduced-order system \( \tilde{M}(s) \) as follows:

\[
\tilde{M}(s) = \Gamma_{22} + \Gamma_{21} X_i (sI - \Lambda_i)^{-1} Y_i^{T} \Gamma_{12}.
\]  
(22)

The characteristic polynomial of the \( \tilde{M} \)-\( \Lambda \) system is now calculated by [17]

\[
P(s, \delta) = \| sI - \Lambda \| \| I - \tilde{M}(s) \|.
\]  
(23)

which yields a polynomial family \( P(s,D) = \{ P(s,\delta) \mid \delta \in D \} \) on the operating domain \( D = \{ \delta \mid \delta_i \in [-1,1], k = 1 \ldots m \} \).

D. Characteristic Polynomial Value Set Calculation

The characteristic polynomial \( P(s,\delta) \) is usually viewed as a polynomial in the variable \( s \); a demanding multi-dimensional grid of \( \delta \) is then called for to study the impact of uncertainty by plotting the polynomial family \( P(s,D) \) in the complex plane. A more practical approach, instead, computes the value set \( P(j(\omega), \delta) \) on a one-dimensional grid of \( \omega \), which not only saves gridding cost, but also allows exploitation of the parameter structure [11].

Since all the uncertain parameters reside in the diagonal matrix \( \Lambda(s) \), we simplify the calculation of \( P(j(\omega), \delta) \) by applying the expansion for diagonal matrices [18] as follows.

Let \( \delta \in \{ \delta_1, \ldots, \delta_m \} \) denote the \((k,k)\)-th element of \( \Lambda \), and \( N \) the size of \( \Lambda \). Evaluate \( P(s,\delta) \) at \( s = j\omega \):

\[
P(j \omega, \delta) = \| j \omega - \delta \| \| \Lambda \|^{-1} \| \tilde{M}(j \omega) \| \]
(24)

\[
S_k = \sum_{\substack{\omega_k \in \omega_0 \cup \omega_1 \cup \cdots \cup \omega_m}} \| \tilde{M}_{i-k}(d_1 \cdots d_{i-k})/(d_1 \cdots d_i) \|
\]

where \( \tilde{M}_{i-k}(d) \) is a matrix formed by removing rows and columns \( i, \ldots, i-k \) of \( \tilde{M}(j\omega) \).

Take a three-parameter case for example: if \( \Lambda = \text{diag}(d_1, d_2, d_3) \) and \( \tilde{M}(j\omega) = (m_{ij}) \in \mathbb{C}^{3 \times 3} \), then

\[
P(j \omega, \delta) = \left( \prod_{k=1}^{3} (j \omega - \lambda_k) \right) (S_0 + S_1 + S_2 + S_3),
\]

where \( \lambda_1, \lambda_2, \lambda_3 \) are the poles of \( \tilde{M}(s) \), and

\[
S_0 = \| \tilde{M}(j\omega) \| d_1 d_2 d_3,
\]

\[
S_i = \left| \begin{array}{ccc}
-m_{i2} & -m_{i3} & m_{i3} \\
-m_{i2} & m_{i3} & -m_{i3} \\
m_{i2} & -m_{i3} & -m_{i3}
\end{array} \right| d_1 d_2 d_3,
\]

\[
S_2 = m_{i3} d_1 - m_{i2} d_2 - m_{i1} d_3,
\]

\[
S_3 = 1.
\]

Note that \( S_k \) contains all the terms of degree \( N-k \) in the characteristic polynomial (24). The coefficient of each term depends on \( \| \tilde{M}_{i-k}(d) \| \), and our experience suggests that, within the frequency range of interest, as \( k \) decreases from \( N \) to 0, the magnitude of the coefficients in \( S_k \) usually drops considerably.

It suffices, therefore, to compute only a part of \( \sum_k S_k \) for the characteristic polynomial: when only \( S_N \) and \( S_{N-1} \) are considered, the parameters \( d_1, \ldots, d_N \) enter (24) in an affine manner, and the edge theorem (theorem 3) can be applied; when more \( S_k \) are computed, the parameters enter (24) in a multilinear manner, and the mapping theorem (theorem 4) is applicable. Nevertheless, it is usually unnecessary to include \( S_k \), \( k \leq N - 3 \), since \( \sum_{k=N-3}^N S_k \) already provides a pretty good approximation, as will be shown in the case study (V-C).

The multilinearity of the polynomial (24), however, is
Obtain state space matrix $A$.

Approximate $A_0$ by $A_0 \approx A_0 + \sum A_i \cdot (\delta_i I)$ (12).

Perform model order reduction of the $M$-$\Delta$ system. (21)

Perform dimension reduction by SVD. (13)-(15)

Calculate characteristic polynomial by diagonal expansion. (24)

Formulate the $M$-$\Delta$ system. (17)-(18)

Apply value set approach.

Fig. 3 The procedure of robustness analysis of large-scale power systems.

negated when there exist repeated parameters in $\Delta$ (i.e., $d_i = d_j$ for some $i, j$). For such a case, transformations can be made to adapt (24) for the mapping theorem (theorem 4), as in [19], by replacing the repeated parameters with new fictitious ones so that the new function is multilinear and consistent with the original one.

IV. RELATED WORKS

In this section, we give a short description of the existing results. Numerical comparisons are provided in the next section.

A. Value Set Construction by Kharitonov theorem

The Kharitonov theorem offers a surprisingly simple stability criterion for interval polynomials, whose value sets are completely determined from only four polynomials.

For an interval polynomial $p(s,a) = a_0 + a_1 s + \cdots + a_n s^n$, $a_i \in [a_i^-, a_i^+]$, the value set $p(j\omega,a)$ can be expressed as $p(j\omega,a) = p_{even} +jp_{odd}$, where $p_{even} = a_0 - a_n \omega^2 + a_{2n} \omega^4 \cdots$ contains the even indexed coefficients, and $p_{odd} = a_0 \omega - a_n \omega^3 + a_{2n} \omega^5 \cdots$ contains the odd indexed ones. Since $p_{even}$ and $p_{odd}$ are mutually independent functions with separate bounds, the value set $p(j\omega,a)$ must be a rectangle in the complex plane, as stated in the following theorem.

Theorem 5 (Kharitonov theorem): For each fixed $\omega \geq 0$, the value set of an interval polynomial $p(j\omega,a)$ is a rectangle with edges parallel to the coordinate axes and with vertices determined by the values of the four Kharitonov polynomials: $p^+(j\omega) = a_0^+ + a_1^+ (j\omega) + a_2^+ (j\omega)^2 + a_3^+ (j\omega)^3 + a_4^+ (j\omega)^4 + \cdots$, $p^- (j\omega) = a_0^- + a_1^- (j\omega) + a_2^- (j\omega)^2 + a_3^- (j\omega)^3 + a_4^- (j\omega)^4 + \cdots$, $p^+(j\omega) = a_0^+ + a_1^+ (j\omega) + a_2^+ (j\omega)^2 + a_3^+ (j\omega)^3 + a_4^+ (j\omega)^4 + \cdots$, $p^- (j\omega) = a_0^- + a_1^- (j\omega) + a_2^- (j\omega)^2 + a_3^- (j\omega)^3 + a_4^- (j\omega)^4 + \cdots$.

The Kharitonov theorem holds only when the coefficients $a_i$ are independent of each other. If they are dependent, however, which is usually the case in power systems, then the value set thus obtained is overconservative and oversimplified.

B. Structured Singular Value Theory

The structured singular value, also denoted as “$\mu$”, gives the smallest size of the uncertainty that causes the system to lose stability. For an $M$-$\Delta$ system as shown in Fig. 2(b), the $\mu$ is defined as

$$\mu(M(j\omega)) = \frac{1}{\min(\sigma_{\text{max}}(\Delta) : |1 - M(j\omega)\Delta| = 0)},$$

where $\sigma_{\text{max}}(\Delta)$ is the maximum singular value. The maximum value of $\mu(M(j\omega))$ over $\omega \in [0, +\infty]$ represents the stability margin. The suggested value set approach complements the $\mu$ theory by revealing the effect of each parameter on stability.

V. CASE STUDY

Several test systems have been used to test the proposed robustness analysis method. A single-machine infinite-bus (SMIB) system is first presented to show the efficacy of the value set approach. A four-machine two-area system is then reported to illustrate the robustness analysis procedure described in III. Finally, we apply the proposed method to the actual North China Grid system.

A. Single-Machine Infinite-Bus System

We start with a SMIB system to give a flavor for the value set approach in the robustness analysis. The test system is described by a sixth order state-space model with a single stage PSS installed at the generator. The parameters of the nominal system are available in [20]. The constant $K_s$, excitation gain $K_{ia}$, and PSS phase compensation parameter $T_1$ are assumed to vary over the following intervals

$$K_s \in [-0.19, -0.09], K_{ia} \in [180, 240], T_1 \in [0.139, 0.185],$$

and are normalized as $\delta_1, \delta_2, \delta_3 \in [-1, 1]$.

Since the order of the state matrix $A$ is small, it is unnecessary to apply the system reduction techniques described in III here, and the characteristic polynomial is directly computed as

$$P(s, \delta) = |sI - A| = c_0 + c_1 \delta_1 + c_2 \delta_2 + c_3 \delta_3 + c_{12} \delta_1 \delta_2 + c_{23} \delta_2 \delta_3, \quad (26)$$

where $c_0 = s^6 + 81 s^5 + 2.3 \times 10^6 s^4 + 2.9 \times 10^8 s^3 + 1.4 \times 10^{10} s^2 + 1.1 \times 10^{12} s + 7.4 \times 10^{14}$, $c_1 = -3.4 s^4 - 110 s - 74) \times 10^3$, $c_2 = (9.8 s^4 + 330 s^3 + 1100 s^2 + 15000 s + 10000) \times 10$, $c_3 = (1.2 s^4 + 62.2 s^3) \times 10$, $c_{12} = (-4.99 s^2 - 150 s - 110) \times 10^2$, $c_{23} = 1.8 s^2 + 89 s^3$.

As $P(s, \delta)$ is multilinear in $\delta_1, \delta_2, \delta_3$, from the mapping theorem (theorem 4), we immediately have the convex hulls of the value sets in the Mikhailov plot (Fig. 4). We then calculate the Mikhailov plot with the edge theorem (theorem 3) (Fig. 5) by truncating the non-affine terms in $P(s, \delta)$—i.e., $c_{12} \delta_1 \delta_2 + c_{23} \delta_2 \delta_3$—since their coefficient magnitudes are relatively small. For comparison, the actual Mikhailov plot is also generated by densely gridding the uncertain parameters $\delta_1, \delta_2, \delta_3$, as shown in Fig. 4.

It is observed that the Mikhailov plot obtained by the mapping theorem is almost identical to the actual plot, due to the simplicity of the polynomial (26). The Mikhailov plot derived from the edge theorem (Fig. 5) consists of a family of parallelograms, which differs, as expected, slightly from the actual plot. Nevertheless, the difference is acceptable considering the computational cost it saves, as shown in the
To examine the impact of the uncertain parameters on system stability, we zoom in on the value set at $\delta_1$, $\delta_2$, $\delta_3 = 0$, we conclude by the zero exclusion theorem (theorem 2) that the system is robustly stable.

To examine the impact of the uncertain parameters on system stability, we zoom in on the value set at $\omega = 7$, agreeing with the frequency of the dominant pole 0.38 $\pm$ 6.9 of the nominal system. The origin is excluded from the value sets at all frequencies, and as the system is stable for the nominal point $\delta_1, \delta_2, \delta_3 = 0$, we conclude by the zero exclusion theorem (theorem 2) that the system is robustly stable.

To examine the impact of the uncertain parameters on system stability, we zoom in on the value set at $\omega = 7.3$ and mark its vertices with the values of $\delta$, e.g., $(+\cdots)$ denotes $\delta_1 = +1, \delta_2 = -1, \delta_3 = -1$, and $(\cdots -)$ denotes $\delta_1 = -1, \delta_2 = +1, \delta_3 = -1$, as shown in Fig. 6(b). We then trace the pairs of vertices with different values of only one parameter. Take parameter $\delta_1$ for example, we locate the pair $(\cdots - +)$ and observe that the increase of $\delta_1$ leads to a southward movement in the Mikhailov plot. The same applies to the pairs $(\cdots + +), (\cdots + -)$, etc., which all conclude the same southward effect of increasing $\delta_1$. Similarly, a southwestward effect is observed by increasing $\delta_2$, and a southward effect is observed by increasing $\delta_3$.

As a result, to improve the stability of the uncertain system, the PSS parameter $T_1$ (that is, $\delta_3$) should be raised to a higher level so that the Mikhailov plot moves farther away from the origin. The higher the $K_p, K_\lambda$ rise, the more stable the system becomes. Because the value set vertex closest to the origin is reached when the three parameters are at their minimum, we infer that the worst case scenario is contributed by the low levels of $K_p, K_\lambda, T_1$, as verified by the eigenvalue analysis in Table I.

For comparison, the Kharitonov theorem (theorem 5) is also employed for the robustness analysis of the test system. The characteristic polynomial (26) is rewritten as an interval polynomial $p(s,a)$ whose coefficients’ bounds are computed separately as in IV-A. The frequency plots of the Kharitonov polynomials $p^{+\cdots}(j\omega), p^{++}(j\omega), p^{++\cdots}(j\omega), p^{+++}(j\omega)$ are then drawn in Fig. 7, where the value sets are expectedly larger than those of the Mikhailov plots in Fig. 4-5, due to the mutual coupling of $p(s,a)$’s coefficients. Because the origin is contained in one of the value sets, we cannot derive the same conclusion from the Kharitonov theorem as from the edge theorem.
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B. Four-Machine Two-Area System

In this section, we use a small-size multi-machine system (Fig. 8) to illustrate the proposed method described in III. The data of the four-machine two-area system is available in [20], whose parameters are shown in Table II. The test system is characterized by a dominant inter-area mode at $-0.17 \pm 3.5$. To analyze this mode, we choose the power export $\exp PSS[0, 400]$ MW, $[8,10]$.

The results of matrix $A$ are shown in Table II, and the generator $G4$ is equipped with a PSS2B [21], whose parameters, which vary over the following intervals:

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$T_{10}$</th>
<th>$T_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.03</td>
<td>0.03</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

To validate the above reduction, we calculate the error between $A$ and the SVD approximation $U_k\Sigma_k V_k^H$ as below.

$$e_k = \left\| A_k - U_k\Sigma_k V_k^H \right\|_F = 0.1\%$$

It is clear that the dimension reduction by the SVD is pretty accurate as long as the dominant singular values are retained in $\Sigma_k$, $k = 1 \ldots m$.

Step 3: Formulate the $M$-$\Delta$ system.

Following the steps in III-B, we have $\Gamma_{11} = [\hat{A}_0]_{30 \times 30}$, $\Gamma_{12} = [U_1\Sigma_{11} U_2\Sigma_{21}]_{30 \times 5}$, $\Gamma_{21} = [V_{11} V_{21}]_{5 \times 30}$, $\Gamma_{22} = 0_{5 \times 5}$, and $\Delta = \text{diag}(\delta_1, \delta_1, \delta_1, \delta_1, \delta_2)_{5 \times 5}$. The transfer function matrix $M(s)$ can be computed by (18), which is of size $5 \times 5$. Note that if we had not performed dimension reduction in step 2, the sizes of $M(s)$ and $\Delta$ would have been $16 \times 16$ or even greater.

Step 4: Perform model order reduction of the $M$-$\Delta$ system.

The order of the above $M$-$\Delta$ system—i.e., the number of eigenvalues in the transfer function matrix $M(s)$—is 30, the same as the size of $\hat{A}_k$. Through the partial fraction expansion, we calculate the reduced-order system $\hat{M}(s)$ by (22), which retains 9 eigenvalues of $M(s)$. Note that although the order of $\hat{M}(s)$ is reduced to 9, its size is unchanged and is still $5 \times 5$.

To validate the model reduction, we compare the dominant eigenvalue of the two systems at different operating conditions, as shown in Fig. 9. The differences of the eigenvalue between the two systems are negligible, hence justifying the reduction.

Step 5: Calculate the characteristic polynomial.

Applying the expansion for diagonal matrices (24), each term of the characteristic polynomial can now be computed and the value set is constructed in turn.

Fig. 10 shows the Mikhailov plot generated by the edge theorem in the frequency range of the dominant mode. As in case A, we mark the vertices of the value set at $\omega = 3.5$ with the
values of $\delta$. We observe that the increase of $\delta_2$ ($K_{pSS}$) contributes to a westward movement within the value set, which pulls the Mikhailov plot farther away from the origin and improves the stability of the mode according to the zero exclusion theorem. The increase of $\delta_1$ ($P_{exp}$) causes a southwestward movement within the value set at $\omega = 3.5$, which seems to have no effect on the envelopment of the origin at that frequency. However, for the value set at $\omega = 3.0$, the southwestward movement orients directly toward the origin and will cause envelopment of the origin for large $\delta_1$ ($P_{exp}$). Therefore, the increase of $P_{exp}$ worsens the mode, and, when instability occurs, the frequency of the mode is lower than before. This conclusion is consistent with the eigenvalue analysis in Fig. 9.

The time-domain simulations of the original system are shown in Fig. 11, where a three-phase fault is applied at one of the tie lines (line 7-8-a) and is cleared in 200 ms. The simulation results confirm the validity of the approach.

C. North China System

1) Case 1: Variations in PSS Parameters

In this section, we apply the proposed method to a North China Grid system with 547 generators, 8647 buses, 3722 lines, 8019 transformers (Fig. 12). The system is characterized by two dominant inter-area modes (Table III) that have grown into a limitation for inter-area power transmission.

A preliminary eigenvalue analysis suggests that some PSSs in Shandong Province actually worsen the modes. Therefore, we choose six parameters of three PSSs in Shandong as the uncertain parameters in order to gain a deeper insight into their impacts.

The PSS model under study is the IEEE PSS2B [21], and the uncertain parameters and their ranges are shown in Table IV. The uncertain parameters $p_1, ..., p_6$ are normalized as $\delta_1, ..., \delta_6 \in [-1, 1]$, and the state space matrix $A$ is of size $7484 \times 7484$.

Since the changes in PSS parameters affect only a small part of $A$’s entries, the dimensions of the $M-A$ system are remarkably reduced to $10 \times 10$ by the SVD (14). The modal truncation of the $M-A$ system is then performed (21) where 140 eigenvalues are retained in the reduced $M-A$ system. With the characteristic polynomial obtained by (24), the Mikhailov plots are generated at $\omega \in [2.05, 2.25]$ so that the frequency range of mode 1 is targeted.

Because the data values of the Mikhailov plot span orders of magnitude, we present the Mikhailov plot on a logarithmic scale, where the magnitude $P(\omega, \delta) = \Re \left( r(\omega, \delta) e^{i \phi(\omega, \delta)} \right)$ is represented by $\log r(\omega, \delta) + \phi(\omega, \delta)$ in polar coordinates, as shown in Fig. 13. Note that the origin “$-\infty$” in Fig. 13 corresponds to the origin “0” on a linear scale, where $\log(0) + \phi(0, \delta) = -\infty$.

We first apply the mapping theorem to obtain the convex hulls of the value sets (Fig. 13) so that the zero exclusion condition (theorem 2) is checked. Then we apply the edge theorem (theorem 3) by truncating the non-affine terms in (24) to produce a Mikhailov plot that approximates the actual one. Such truncation of the non-affine terms is supported by the fact that

![Fig. 12 Simplified single-line diagram of the North China system.](image)

![Fig. 13 The Mikhailov plot derived from the mapping theorem on the logarithmic scale, with no inclusion of the origin. (case C-1)](image)
that the coefficient magnitudes of the higher-degree terms are much lower than those of the affine terms, as shown in Table V.

Now we examine the parameters’ impact on mode 1 by tracing the movements of image $P(\omega, \delta)$ in the Mikhailov plot. Fig. 14(a)-(f) show the Mikhailov plots on a linear scale with the vertex images at $\omega = 2.15$ marked. The vertex images here refer to the images of the vertices of $D = \{ \delta | \delta_k \in [-1,1], k = 1\ldots6 \}$, and the movement of each vertex image is drawn by increasing one of the parameters $\delta_i$ from $-1$ to $+1$ while fixing the other parameters $\delta_i$, $i \neq k$ at their minimum or maximum.

<table>
<thead>
<tr>
<th>Term</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$d_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>1</td>
<td>0.08</td>
<td>0.01</td>
<td>0.3</td>
<td>0.08</td>
</tr>
<tr>
<td>Magnitude</td>
<td>0.2</td>
<td>5\times10^{-4}</td>
<td>1\times10^{3}</td>
<td>0.03</td>
<td>4\times10^{4}</td>
</tr>
</tbody>
</table>

*Table V: Some of the Coefficient Magnitudes of the Characteristic Polynomial $P(\omega, \delta)$ at $\omega = 2.15$ in Case C-1*

![Fig. 14](image_url)

Fig. 14 The movements of image $P(\omega, \delta)$ at $\omega = 2.15$ on the complex plane as (a) $\delta_1$, (b) $\delta_2$, (c) $\delta_3$, (d) $\delta_4$, (e) $\delta_5$, (f) $\delta_6$ increases from $-1$ to $+1$, on a linear scale plot.

![Fig. 15](image_url)

Fig. 15 The Mikhailov plot derived from the mapping theorem on the logarithmic scale, with no inclusion of the origin. (case C-2)

Now we examine the parameters’ impact on mode 1 by tracing the movements of image $P(\omega, \delta)$ in the Mikhailov plot.

![Fig. 14](image_url)

### 2) Case 2: Variations in Power Transfer Levels

The test system used in this section is the same as that of the previous section, except that the uncertain parameters are the power transfers across three major interfaces, as shown in Table VI. The increased transfer levels across these three interfaces are known to have an adverse effect on mode 1, so here we apply the value set approach to delve deeper into their effects.

The uncertain parameters $p_1$, $p_2$, $p_3$ are normalized as $\delta_1$, $\delta_2$, $\delta_3 \in [-1,1]$, and the dimensions of the $\mathbf{M}$-$\mathbf{A}$ system are reduced to $38\times38$ by the SVD (14). 114 eigenvalues are retained in the $\mathbf{M}$-$\mathbf{A}$ system by the modal truncation (21), and the Mikhailov plot is generated at $\omega \in [2.05,2.25]$, as shown in Fig. 15.

<table>
<thead>
<tr>
<th>Description</th>
<th>Power transfer from Central China to Shaanxi.</th>
<th>Power transfer from Shandong to South Hebei</th>
<th>Power transfer across the Inner Mongolia interface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (MW)</td>
<td>$[-6225.5, -5225.5]$</td>
<td>$[151.4, 1151.4]$</td>
<td>$[7924.9, 8924.9]$</td>
</tr>
</tbody>
</table>

![Fig. 15](image_url)

Fig. 15 The Mikhailov plot derived from the mapping theorem on the logarithmic scale, with no inclusion of the origin. (case C-2)
As \( \omega \) increases, the convex hull rotates around the origin and reaches its maximum size at around \( \omega = 2.15 \)—the same frequency of mode 1. This is not a coincidence, since the size of the value set reflects the influence of the parameters and the value set at the mode frequency is more liable to expand and envelop the origin.

Fig. 16(a)-(c) trace the movements of image \( P(j\omega,\delta) \) at \( \omega = 2.12, 2.15, 2.18 \) on the complex plane as (a) \( \delta_1 \), (b) \( \delta_2 \), (c) \( \delta_3 \) increases from \(-1\) to \(+1\), on a linear scale plot.

As \( \omega \) increases, the convex hull rotates around the origin and reaches its maximum size at around \( \omega = 2.15 \)—the same frequency of mode 1. This is not a coincidence, since the size of the value set reflects the influence of the parameters and the value set at the mode frequency is more liable to expand and envelop the origin.

Fig. 16(a)-(c) trace the movements of image \( P(j\omega,\delta) \) at \( \omega = 2.12, 2.15, 2.18 \) as in case C-1. A rotational symmetry is observed for the movements within different value sets. It is clear that the increase of \( \delta_1 \) or \( \delta_2 \) contributes to an inward movement to the origin, whereas the increase of \( \delta_3 \) produces an almost tangential movement. For \( \delta_3 \), the tangential movements first approach the origin and then depart from it. Specifically, if \( \delta_1 \) and \( \delta_2 \) are increased so that \( P(j\omega,\delta) \) is near the origin, then the tangential movement of \( \delta_3 \) may cause an envelopment of the origin and leads to instability.

As a result, all the three parameters conspire to drag the value set towards the origin and worsen the mode 1 according to the zero exclusion theorem (theorem 2). To improve the system stability, one should limit the three transfer levels. This result is consistent with the operating experience of North China utility engineers.

3) Case 3: Variations in Load Levels

In this study, we choose the load level of Beijing \( P_{BJ} \) and the load level of Inner Mongolia \( P_{IM} \) as the uncertain parameters, which vary over the following intervals:

\[
P_{BJ} \in [16765,16865] \text{ MW}, \quad P_{IM} \in [18700,18800] \text{ MW}.
\]

The dimensions of the reduced \( \mathbf{M} - \mathbf{A} \) system is 17\( \times \)17, and 120 eigenvalues are retained in the reduced \( \mathbf{M} - \mathbf{A} \) system. The Mikhailov plot is generated at \( \omega \in [2.13,2.16] \) as shown in Fig. 17. Fig. 18(a)-(b) trace the movements of image \( P(j\omega,\delta) \) as \( P_{BJ} \), \( P_{IM} \) increase respectively.

We observe that the effect of increasing \( P_{IM} \) on the image movements is greater than that of increasing \( P_{BJ} \). For \( P_{IM} \), the outward movements of the images pull the Mikhailov plot farther away from the origin and therefore improves the stability of the mode. For \( P_{BJ} \), the image movements at lower frequencies (\( \omega < 2.14 \)) are tangential to the origin and will not affect the envelopment of the origin for those value sets. At higher frequencies (\( \omega > 2.14 \)), the image movements produced by \( P_{BJ} \) become far less significant than that produced by \( P_{IM} \). Therefore, we conclude that the increased load level of Inner Mongolia \( P_{IM} \) will improve the stability of mode 1, whereas the load level of Beijing \( P_{BJ} \) contributes little to the mode’s stability. These findings agree with the eigenvalue analysis of the original system.

In the above case studies, we have presented robustness analysis of large power systems for as many as six uncertain parameters. Analysis of more uncertain parameters is possible. Nevertheless, in power systems, such a moderate number of uncertain parameters is often sufficient for engineering applications. The method can be further developed to accommodate higher numbers of parameters. Case studies of more than 50 uncertain parameters are reported in [11].

VI. CONCLUSION

This paper presents a practical method for analyzing the impact of uncertain parameters on small-signal stability of large-scale power systems. The uncertain system is first
reduced and modelled in an \( M \Delta \) framework, from which the characteristic polynomial is obtained through a diagonal expansion. The Mikhailov plot is then drawn by applying the edge theorem and mapping theorem. By tracing the image movement in the plot, the impact of uncertainty is revealed. The proposed method applies to a wide range of parameter variations, and can unravel the coupling of the parameters. The Kharitonov theorem and the \( \mu \) theory are also tested in the study, and the results of Kharitonov theorem are reported.

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Jinghao Zhou received his B.S. degree in electrical engineering from the School of Electrical Engineering, Zhejiang University, Hangzhou, China, in July 2013. He is currently working towards the Ph.D. degree in the School of Electrical Engineering, Zhejiang University, China. His research interests include power system stability analysis and control.

Peng Shi received his B.S. degree in electrical engineering from the School of Electrical Engineering, Zhejiang University, Hangzhou, China, in July 2014. He is currently working towards the Ph.D. degree in the School of Electrical Engineering, Zhejiang University, China. His research interests include power system stability analysis and control.


Ying Xu received his B.S., M.S., and Ph.D. degrees in power system engineering from Harbin Institute of Technology, Harbin, China, in 2003, 2005, and 2009, respectively. Currently, he is working as a Senior Engineer with North China Branch, State Grid Corporation of China, Beijing, China. His research interests include power system stability control and analysis, renewable energy and its integration into power grid, energy market reform, and policy making in China.

Huanhai Xin was born in Jiangxi, China, in 1981. He received the Ph.D. degree from the Department of Electrical Engineering, Zhejiang University, Hangzhou, China, in June 2007. He is currently a Professor in the Department of Electrical Engineering, Zhejiang University. He was a post-doctor in the Electrical Engineering and Computer Science Department of the University of Central Florida, Orlando, from June 2009 to July 2010. His research interests include power system stability analysis and renewable energy integration.

Changming Jiang received the M.S. degree in power system engineering from Chongqing University, Chongqing, China, in 1996. Currently, he is working as a Senior Engineer with North China Branch, State Grid Corporation of China, Beijing, China. His research interests include power system control and analysis.

Huan Xie received the Ph.D. degree from the Xi’an Jiao Tong University, Xi’an, China, in 2008 and the M.Tech degree in electrical power systems form Ho Hai University, Nan Jing, China, in 2004. He is currently a senior engineer in North China Electric Power Research Institute Co., Ltd. His research interests include power system stability, adaptive control, and Automatic voltage regulator.

Tao Wu was born in Beijing, China, in 1968. He received the Ph.D. degree from the Xi’an Jiao Tong University, Xi’an, China, in 1997. He is currently a professorial senior engineer in North China Electric Power Research Institute Co., Ltd. His research interests include power system stability, renewable energy integration, and Automatic voltage regulator.