Assessing Strength of Multi-infeed LCC-HVDC Systems Using Generalized Short Circuit Ratio

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Abstract—In this paper, a generalized short circuit ratio (gSCR) is proposed for assessing the grid strength of multi-infeed LCC-HVDC (MIDC) systems by the theoretical analysis of the relationship between static voltage stability and grid strength. The gSCR can resolve the ambiguity when applying the concept of short circuit ratio (SCR) for single-infeed LCC-HVDC (SIDC) systems to MIDC systems. Moreover, the gSCR is the generalized presentation of SCR in MIDC systems. Based on the gSCR, our defined critical gSCR (CgSCR) and boundary gSCR (BgSCR) for MIDC systems have the same functions as the critical SCR (CSCR) and boundary SCR (BSCR) used in grid strength analysis for SIDC systems. Also, the CgSCR and BgSCR have the same values as those of CSCGR and BSCR (i.e., 
\[
\text{CgSCR} = \text{CSCR} = 2 \quad \text{and} \quad \text{BgSCR} = \text{BSCR} = 3
\]
) to characterize the stability boundaries of MIDC systems. This allows the analysis experience of using the SCR, CSCR, and BSCR for SIDC systems can be easily transplanted to the application of the gSCR, CgSCR and BgSCR for MIDC systems. The efficacy of the proposed gSCR along with CgSCR and BgSCR is demonstrated by comparisons them with other indices of grid strength assessment for MIDC systems.

Index Terms—static voltage stability, multi-infeed LCC-HVDC systems, generalized short circuit ratio, CgSCR and BgSCR.

I. INTRODUCTION

The increasing use of line commutated converter-based high voltage direct current (LCC-HVDC) brings technical and economic advantages for long-distance and high-capacity transmission [1-3]. Particularly, in China, LCC-HVDC transmission projects have led to the configurations called multi-infeed LCC-HVDC (MIDC) systems (hereinafter, the LCC-HVDC MIDC system is called the MIDC system) jointly with ultra-high-voltage ac transmissions, which will bring the total installed capacity of HVDCs exceeding 700GW by 2020 [2][3]. On the other hand, when LCC-HVDCs are connected to ac power grids with low strength and the high impedances (i.e., weak ac grids), it may cause potential stability issues such as temporary overvoltage, concurrent commutation failure and the maximum available power (MAP) that is less than the rated transmission power [4-6].

The short circuit ratio (SCR) has been extensively used to assess the grid strength of single-infeed LCC-HVDC (SIDC) systems (hereinafter, the LCC-HVDC SIDC system is called the SIDC system) with exact boundaries [7]. However, the straightforward application of the concept of SCR for SIDC systems to MIDC systems has caused certain ambiguity since the impact of the interactions among HVDC inverters cannot be reflected at a referred inverter ac bus [8]. To take into account the interactions, various SCR-based methods have been presented in [8, 9]. In [8], the multi-infeed interactive short circuit ratio (MSCR) was defined by considering the impact of interactions using the multi-infeed interaction factor (MIIF), which is the ratio of change of voltages at HVDC inverter buses. In [9], the multi-infeed short circuit ratio (MSCR) was defined by considering the impact using the impedance ratio.

However, these MSCR and MSCR have the following shortcomings: 1) These MSCR and MSCR are derived from empirical or heuristic reasoning and short of theoretical justification. 2) Only the critical MSCR (CMSCR) and critical MSCR are defined to study the transfer limit or the voltage stability, and the CMSCR and critical MSCR may be variable for different power systems. For example, for one system the CMSCR is 2.0, but the value is changed to be 1.4 for another system. Consequently, even when the MSCR or MSCR are calculated, it is difficult to know where the stability margin is (or whether the system is strong or weak), which means these two indices cannot be used conveniently. 3) Only the critical location can be identified by MSCR and MSCR, while the critical direction of power change cannot be provided.

Among these shortcomings, only shortcoming 1) was overcome in [4, 10-12]: some indices were theoretically proposed by investigating the singularity point of the Jacobian matrix in [4, 10-11], but they focused on the characteristic analysis of the Jacobian matrix and there was no such a SCR proposed as in SIDC; in [12] the equivalent effective short circuit ratio (EESCR) was proposed by the equivalent analysis, but it needs detailed system operation data, which may not be available easily in the MIDC systems [12].

To address the above issues, in this paper the generalized short circuit ratio (gSCR) is proposed for assessing the grid strength of MIDC systems. Compared to MSCR and MSCR, the gSCR has the following advantages:

a) The gSCR has a rigorous definition based on static voltage stability analysis. The gSCR is derived by the theoretical analysis of the relationship between SCR and static voltage stability in SIDC systems and extending the analysis results to MIDC systems based on modal analysis. This analysis technique was used for static voltage stability analysis in power systems [13].

b) The gSCR is the generalized presentation of SCR in MIDC systems and it has the same physical meaning of SCR. Based on gSCR, the critical gSCR (CgSCR) and boundary gSCR (BgSCR) can be analogously defined and then have the same
functions of critical SCR (CSCR) and boundary SCR (BSCR) used for grid strength assessment in SIDC systems. Moreover, the CgSCR and BgSCR have the same values of CSCR and BSCR, i.e., CgSCR=CSCR=2 and BgSCR=BSCR=3. Thus, the analysis experience of using the SCR, CSCR, and BSCR for SIDC systems can be transplanted to the application of the gSCR, CgSCR and BgSCR in MIDC systems, which can simplify the complex stability analysis of MIDC systems.

c) Based on gSCR, not only the critical location can be identified, but also the critical direction of power change can be provided to identify the nearest instability point and the stability boundary.

The rest of this paper is organized as follows: In Section II, the relationship between the static voltage stability and grid strength of SIDC systems is analyzed. Then, the gSCR is defined by extending the relationship to MIDC systems in Section III. Section IV shows the main characteristics of gSCR compared with the other indices. The validity of the proposed gSCR is verified by numerical simulations in Section V and the conclusions are drawn in Section VI.

II. RELATIONSHIP BETWEEN STATIC VOLTAGE STABILITY AND GRID STRENGTH IN SIDC SYSTEMS

A. Static Voltage Stability Analysis in SIDC Systems

A SIDC system can be represented with a classical model as shown in Fig.1, where the inverter end in this system is represented by the Thevenin source E and Thevenin impedance Z and the control mode of the HVDC is constant current constant extinction angle (CC-CEA) or constant power constant extinction angle (CP-CEA) [14].

![Fig. 1 Single-infeed LCC-HVDC system](image)

In this SIDC system, the power flow equations can be expressed as [14],

\[
\begin{align*}
    P_d &= U_d I_d \\
    U_d &= 3\sqrt{2}\pi^3 N K U \cos \gamma - 3\pi^3 N X I_d \\
    \cos \phi &= \cos \gamma - X I_d / \sqrt{2} K U \\
    Q_d &= -P_d \tan \phi \cos \delta / Z \\
    Q &= Q_d - (U^2 - U E \cos \delta) / Z \\
\end{align*}
\]

where \( P \) and \( Q \) are the active and reactive power injected into the inverter-end ac bus by HVDC and ac system; \( P_d, I_d \) and \( U_d \) are the DC power, current and voltage, respectively; \( Q_d \) is the reactive power of the HVDC together with the compensation capacitor; \( R \) is the DC resistance, which is neglected here; \( K \) is the ratio of transformer; \( U \) is the AC voltage; \( X \) is commutation reactance; \( \gamma \) is the extinction angle; \( N \) is the number of bridge converters; \( \delta \) is the voltage angle between \( E \) and \( U \); \( \phi \) is the power-factor angle of the inverter; \( B \) is the reactive power compensation capacitor; \( \omega \) is the angular velocity.

Based on (1), the linearized power flow equations at the inverter side can be written as

\[
\begin{bmatrix}
    \Delta P_d \\
    \Delta Q_d \\
    \Delta U_d \\
    \Delta \gamma \\
\end{bmatrix} = J_{sdc} \begin{bmatrix}
    \Delta I_d \\
    \Delta \phi \\
    \Delta \delta \\
    \Delta \psi \\
\end{bmatrix}
\]

In (2), DC current \( I_d \), ac voltage magnitude \( U \) at the inverter side and phase angle \( \delta \) at the inverter AC bus are chosen as state variables; \( \Delta I_d, \Delta \phi, \Delta \delta \) and \( \Delta U_d / U \) represent the perturbations of current \( I_d \), phase angle \( \delta \) and voltage \( U \) in percentage; \( \Delta P_d, \Delta P \) and \( \Delta Q \) represent the perturbations of the DC power \( P_d \), active power \( P \) and reactive power \( Q \) injected into the inverter-end ac bus by HVDC and ac system; \( J_{sdc} \) is the Jacobian matrix, which can be further represented as

\[
J_{sdc} = \begin{bmatrix}
    J_{p,d} & J_{p,\phi} & J_{p,\psi} \\
    J_{p,\phi} & J_{\phi,\phi} & J_{\phi,\psi} \\
    J_{p,\psi} & J_{\phi,\psi} & J_{\psi,\psi} \\
\end{bmatrix}
\]

where elements in matrix \( J_{sdc} \) are partial derivatives of the power-flow equations with respect to variables \( I_d, \phi \) and \( U \). The elements \( J_{p,d}\phi=0 \) and \( J_{Q,d}\phi=0 \) since the DC power \( P_d \) and the reactive power injected into the inverter-end ac bus by HVDC \( Q_d \) are insensitive to the phase angle \( \delta \) according to the characteristics of LCC-HVDC transmission; the other elements in matrix \( J_{sdc} \) can be represented as

\[
\begin{align*}
    J_{p,d} &= U_d - 3\pi^3 N X I_d \\
    J_{p,\phi} &= (U_d + 3\pi^3 N X I_d) I_d \\
    J_{p,\psi} &= -P_d \tan \phi + P_d K(c) + 2\omega B U^2 \\
    J_{Q,d} &= (U_d - 3\pi^3 N X I_d) \tan \phi - P_d K(c) / I_d \\
    J_{Q,\phi} &= J_{p,\phi} - P_d U^2 - Z \\
\end{align*}
\]

\[
K(c) = \frac{1}{[(\cos \gamma - \delta)^2 \sqrt{1 - (\cos \gamma - \delta)^2} ]}, \quad c = XI_d / (\sqrt{2} K U) .
\]

The detailed derivations are provided in Appendix A. And the reactive power absorbed by HVDC is assumed to be compensated locally by compensation capacitor, i.e., \( \tan \phi = \omega B U^2 / P_d \).

It is known that static voltage instability (i.e., the saddle-node bifurcation) would occur in the system only if the Jacobian matrix \( J_{sdc} \) in (3) becomes singular [4, 10-11]. That is, the boundary condition for the static voltage stability is such that the determinant of the matrix \( J_{sdc} \) is equal to zero. It is equivalent to

\[
\det(J_{sdc}) = \det(J_{p,d}) \det(J_{\phi,\phi}) = 0
\]

where

\[
J_{\phi,\phi} = \begin{bmatrix}
    -U^2 Z^{-1} & -P_d \\
    -U^2 Z^{-1} & -P_d & \gamma_d \\
\end{bmatrix}
\]

Since the determinant of matrix \( J_{p,d} \) is nonzero, (5) is equivalent to

\[
\det(J_{sdc}) = \det(J_{\phi,\phi}) = 0
\]

In (6), \( J_{p,d} \) can be further represented with (8) below by substituting (4) into (6),
\[
J_{Q_{1u}} - J_{Q_{1l}} J_{P_{2l}}^{-1} J_{Q_{2l}} = 2P_c \cos(\theta) \left[ 1 - \left( \frac{\cos \gamma}{c} - 1 \right) \right] + 2 \omega B_z U_z^2 \\
= P_N U_z^2 \rho T
\]

where \( T = 2c \cos(\theta) \left[ 1 - \left( \frac{\cos \gamma}{c} - 1 \right) \right] + 2 \omega B_z U_z^2 / \rho_d = \rho_p (P_N U_z^2) \), and \( P_N \) is the transmitted rated power of HVDC.

By using Schur decomposition, (7) can be rewritten as,
\[
\det(J_{S_{IDC}}) = \det(J_{S_{sys}})
\]
\[
= \det(-U^T Z^{-1}) \det(-U^T Z^{-1} + P_N U^2 \rho T + P_N^2 Z U^{-2}) = 0
\]

Equation (9) is equivalent to
\[
\rho T + \rho P_N Z = 1/\rho P_N = 0
\]

In (10), \( T \) and \( \rho \) are related to the operation point of HVDC. It can be proved that \( T \) and \( \rho \) are constant values at a given specific operation point. The proof can be found in Appendix B. Thus, (10) can be rewritten as,
\[
1/\rho P_N Z = \rho T / 2 + \sqrt{\rho T^2 / 4 + \rho^2}
\]

Alternatively, (11) becomes the boundary condition of static voltage stability from (5) at the specific operation point of HVDC, where \( T \) and \( \rho \) are constant values.

### B. Relationship between Static Voltage Stability and Grid strength

In a SIDC system, the grid strength at an inverter end ac bus can be evaluated with SCR, which is the ratio of the short circuit capacity at an inverter terminal ac bus to the rated HVDC power injected into this ac bus [7].

\[
SCR = \frac{S_{ac}}{P_{ac}} = \frac{U_n^2}{P_{ac}} \times \frac{1}{Z} = \frac{1}{P_{ac} Z}
\]

where \( S_{ac} \) is the short circuit capacity of ac system; \( P_{ac} \) is the rated HVDC power; and \( U_n \) is the rated ac voltage.

Replacing \( 1/\rho P_N Z \) with SCR in (11) yields,
\[
SCR = \rho T / 2 + \sqrt{\rho T^2 / 4 + \rho^2}
\]

Similar to (11), (13) becomes the boundary condition of static voltage stability. Thus, SCR has a straight relationship with the static voltage stability.

Moreover, based on the SCR defined in (13) and the operation point at the static voltage stability boundary, another two indices can be further introduced, i.e., critical SCR (CSCR) and boundary SCR (BSCR) [7]:

1) CSCR is the value of SCR when the MAP point of HVDC coincides with its rated operation point. CSCR can be used to distinguish a very weak system from a weak system. The ac system is very weak means that the HVDC cannot transfer the rated power stably.

2) BSCR is the value of SCR when the MAP point of HVDC coincides with its operation point of commutation overlap angle (COLA) equal to 30° (the COLA is required to be less than 30° in 12-pulse bridge converters [14]). BSCR can be used to distinguish a weak system from a strong system. The ac system is strong means the HVDC transmission power is only limited by the COLA [14].

The above conclusions are summarized in Fig. 2. Namely, when SCR is smaller than the CSCR, the ac system is very weak and the HVDC cannot transmit rated power stably. When SCR is larger than the BSCR, the ac system is strong. For a real HVDC system, CSCR and BSCR are close to 2 and 3, respectively [7].

The following work of this paper is to extend the conclusions in Fig. 2 from SIDC systems to MIDC systems by defining a new index so-called gSCR.

### III. PROPOSED GSCR FOR GRID STRENGTH ASSESSMENT IN MIDC SYSTEMS

#### A. Static Voltage Stability Analysis in MIDC Systems

Consider a MIDC system shown in Fig. 3, where multiple SIDCs are inter-connected by coupling impedances [8].

![Fig. 3 Multi-infeed LCC-HVDC system](image)

The impedances and Thevenin impedances mentioned can be found from the standard data sets used in system studies, which has been clarified in [12, 15-16]. In addition, the business software PSD-BPA can also be used to directly calculate the impedances from the standard data sets used in system studies.

To simplify the analysis, the following assumptions are considered in the MIDC system:

**Assumption 1.** The HVDCs are similar, i.e., their extinction angles (EAs), COLAs are same and per-unit value parameters based on their individual power capacity are all same. The control modes of all HVDCs are CP-CEA or CC-CEA.

**Assumption 2.** The topology of the ac system is strongly connected and inductive. In other words, the resistances of interconnection lines between HVDC and the ac system can be ignored, and the node admittance matrix that only retains HVDC nodes is reversible and symmetrical.

**Assumption 3.** The active power on interconnection lines between HVDCs is smaller than their transmission limits.

The following remarks are given for these assumptions:

1) Assumption 1 can be generally satisfied for actual systems. For example, in China, the HVDCs manufactured by the same manufacturer usually operate under the same control strategies (the constant extinction angle control) and use same per-unit value parameters normalized based on their individual power...
capacity. In addition, as shown later, our work focus on the stability under two important operating points: the rated operation point and the operating point where the COLAs of all HVDCs are 30°. Under both these two point, the COLAs of all HVDCs are same as well.

2) Assumption 2 and assumption 3 are generally valid since the HVDCs are designed to transfer active power and connected to ultra-high voltage transmission networks such that the line reactances are much larger than the line resistances and active power interchange among the HVDCs are small. These two assumptions will be further discussed in our simulations by calculating the sensitivity of gSCR with respect to these two assumptions. The results show that these assumptions can be relaxed and gSCR can still be used to assess the MIDC systems with exact values when these assumptions are not fully satisfied.

Similar as (2), the linearized power flow equations at the inverter side in the MIDC systems can be represented in vector form as follows,

$$\begin{bmatrix} \Delta P_d \\ \Delta P \\ \Delta Q \end{bmatrix} = J_{MIDC} \begin{bmatrix} \Delta L_d \\ \Delta \delta \\ \Delta U/U \end{bmatrix}$$

(14)

where $\Delta P_d$, $\Delta P$ and $\Delta Q$ are the vectors representing the perturbations of the DC power, active and reactive power at each inverter ac bus, respectively; $\Delta L_d$, $\Delta \delta$ and $\Delta U/U$ are the vectors representing the perturbations of the DC currents, phase angle at each inverter ac bus and the ac voltage percentage; $J_{MIDC}$ is the Jacobian matrix and represented as:

$$J_{MIDC} = \begin{bmatrix} J_{P,i} & J_{P,s} & J_{P,U} \\ J_{Q,i} & J_{Q,s} & J_{Q,U} \end{bmatrix}$$

(15)

where $J_{P,i}$, $J_{Q,i}$, $J_{P,s}$ and $J_{Q,s}$ are all diagonal matrices, whose elements are represented in (4). It is worth mentioning that, $J_{P,i}$ is a diagonal matrix does not mean that the HVDCs do not interact with each other in the MIDC systems, e.g., the $P_2$ of HVDC1 can be changed if the $I_2$ of HVDC2 changes. It is more visualized in (31), where $J_{eq}$ is not diagonal. Similarly, $J_{Q,i} = 0$, $J_{P,s} = 0$. $J_{Q,s}$, $J_{P,U}$ and $J_{Q,U}$ are system partial derivatives matrices, which can be represented as [17],

$$J_{P,s} = B \text{diag}(U_1^{1/2}) \quad J_{P,U} = - \text{diag}(P_i) \quad J_{Q,U} = - \text{diag}(P_i) \quad J_{Q,s} = B \text{diag}(U_1^{1/2}) + J_{Q,U}$$

(16)

where diag($a_{ij}$) represents diagonal matrix diag($a_{11}, a_{22}, \ldots, a_{nn}$); $P_i$ is the active power output of the $i$th HVDC; $U_i$ is the ac terminal bus voltage of the $i$th inverter; and $B$ is the node susceptance matrix.

It is known that when the Jacobian matrix $J_{MIDC}$ in (15) becomes singular, static voltage instability (i.e., the saddle-node bifurcation) would occur in MIDC systems [4, 10-11]. That is, the boundary condition for the static voltage stability is such that the determinant of the matrix $J_{MIDC}$ is equal to zero.

$$\det(J_{MIDC}) = 0$$

(17)

In (17), the determinant of the Jacobian matrix $J_{MIDC}$ can be decomposed as,

$$\det(J_{MIDC}) = \det(J_{P,i}) \det(J_{Q,i}) = 0$$

(18)

In (18), the determinant of matrix $J_{P,i}$ is nonzero. Thus, when the determinant of matrix $J_{MIDC}$ is zero, the determinant of matrix $J_{eq}$ is zero. That is, the boundary condition for the static voltage stability is such that the determinant of the matrix $J_{eq}$ is equal to zero.

$$\det(J_{eq}) = \det \begin{bmatrix} B \text{diag}(U_1^{1/2}) & - \text{diag}(P_i) \\ - \text{diag}(P_i) & B \text{diag}(U_1^{1/2}) + J_{Q,U} \end{bmatrix} = 0$$

(19)

In (19), $J_{Q,U} - J_{Q,U}^{-1} J_{P,U}$ can be further represented with below by substituting (4) into (19) and assuming that the reactive power absorbed by HVDCs is compensated locally by compensation capacitors, i.e., $\tan\beta = \omega B_i U_i^2 / P_i$

$$J_{Q,U} - J_{Q,U}^{-1} J_{P,U} = 2 P_i/\omega K_i \left[1 - \frac{1}{\cos^2 \gamma} \right] + 2 \omega B_i U_i^2$$

(20)

Here, $T_i = 2 \omega K_i \left[1 - \frac{1}{\cos^2 \gamma} \right] + 2 \omega B_i U_i^2 / P_i$. If $P_i = P_i \omega / U_i^2 T_i$. In (20), $T_i$ and $\rho_i$ are related to the operation point of the $i$th HVDC. It can be proved that for any $i$ in a MIDC system, $T_i$ and $\rho_i$ are constant and equal are at a given specific operation point, i.e., $\rho_1 = \rho_2 = \ldots = \rho_n = \rho$ and $T_1 = T_2 = \ldots = T_n = T$. The proof can be found in Appendix B.

By using Schur decomposition, (19) can be rewritten as,

$$\det(J_{eq}) = \det(J_{eq}) = \det \left[ B \text{diag}(U_1^{1/2}) \cdot \text{diag}\left(\frac{P_i U_i^2 \rho T}{2}\right) - \text{diag}(P_i) \right] = 0$$

(21)

Since $\det(B \text{diag}(U_1^{1/2})) \neq 0$, we rewrite (21) by left-multiplying diag$^{-1}(P_i)$ $U_i^2$ as,

$$\det \left[ \rho T \cdot 1 + \rho^2 J_{eq}^{-1} J_{eq} \right] = 0$$

(22)

where $J_{eq}$ is defined as the extended Jacobian matrix

$$J_{eq} = -DB$$

(23)

where $D = \text{diag}^{-1}(P_n)$ and $P_n$ is the rated transmission power of HVDC injected into the $n$th terminal bus. Some properties on $J_{eq}$ are presented in Appendices C and D.

It can be proved that there exists a nonsingular matrix $W$, such that the eigenvalues of $J_{eq}$ can be sorted as $0 < \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_n$ by ascending order.

$$W^{-1} J_{eq} W = \text{diag}\left\{\lambda_1, \lambda_2, \ldots, \lambda_n\right\}$$

(24)

Thus, (22) and (24) yields,

$$\prod_{i=1,2,\ldots} (\rho T + \rho^2 \lambda_i - \lambda_i) = 0$$

(25)

It can be seen from (25) that the static voltage stability boundary can be mathematically described by a specific $\lambda_i$ that meet the condition of (25). Naturally, the minimum eigenvalue determines the result, so (25) is simplified to the following equation

$$\rho T + \rho^2 \lambda_1 - \lambda_1 = 0$$

(26)

where $\lambda_1$ represents the minimum eigenvalue of matrix $J_{eq}$ defined in (28). As shown in (19), $T$ and $\rho$ in (26) are related to the operation point of HVDC, and they are constant values at a given specific operation point, which is proved in Appendix B. Thus, (26) can be rewritten as,
\[ \lambda_1 = \rho T / 2 + \sqrt{\rho^2 T^2 / 4 + \rho^2} \]  

(27)

Equation (27) indicates that \( \lambda_1 \) is the condition of static voltage stability boundary at the specific operation point of HVDC, when \( T \) and \( \rho \) are constant values. Thus, \( \lambda_1 \) has a straight relationship with the static voltage stability.

**B. Proposed gSCR in the MIDC Systems**

Comparing (27) with (13), it can be seen that \( \lambda_1 \) plays the same role in MIDC systems as SCR in SDC systems. Motivated by this observation, we define gSCR based on \( \lambda_1 \) in MIDC systems.

**Definition 1**: The minimum eigenvalue of the extended Jacobian matrix \( J_{eq} \) is defined as the gSCR, i.e.,

\[ gSCR = \min(\lambda(J_{eq})) \]  

(28)

Based on the gSCR in MIDC systems, we further define critical gSCR (CgSCR) and boundary gSCR (BgSCR), which are similar to the SCR in SDC systems and BSCR defined in SDC systems. As mentioned above in Section II, in SDC systems, SCR and BSCR are defined based on the relationship between the transmission power and current at the specific operation point. However, in MIDC systems, there are at least two HVDCs, the relationship between the transmission power and current at the static voltage stability boundary becomes much more difficult to be analyzed. When a MIDC system reaches its static voltage stability boundary (the saddle-node bifurcation), the HVDCs’ power is incompletely controllable, i.e., there does not exist a current change direction where all HVDCs’ power increases. The conclusion is derived after Definition 3. Thus, the CgSCR and BgSCR can be defined as follows:

**Definition 2**: CgSCR is the value of the gSCR when the HVDCs’ power is incompletely controllable at the rated operation point. Like SCR in SDC systems, the CgSCR in MIDC systems can be used to distinguish a very weak system from a weak system. The CgSCR in MIDC systems is equal to the SCR in SDC systems, i.e., CgSCR = SCR = 2, since \( \rho \) and \( T \) in (13) and (27) are all same in SDC and MIDC systems at the rated operation point. Thus, when gSCR is smaller than CgSCR, the ac system of the ac system is very weak, and the HVDCs cannot transmit rated power stably.

**Definition 3**: BgSCR is the value of the gSCR when the HVDCs’ power is incompletely controllable at the operation point under which COLAs are equal to 30° for all HVDCs. Similar to BSCR in SDC systems, the BgSCR in MIDC systems can be used to distinguish a weak system from a strong system. The BgSCR in MIDC systems is equal to BSCR in SDC systems, i.e., BgSCR = BSCR = 3. Since, \( \rho \) and \( T \) in (13) and (27) are all same in SDC and MIDC systems at the operation point under which COLAs are equal to 30°. Thus, when gSCR is smaller than BSCR but larger than CgSCR, the ac system of the ac system is weak. The HVDCs’ transmission power is not limited by their own transmission capability, but limited by the static voltage stability. When gSCR is larger than BgSCR, the ac system in the MIDC system is strong, and the HVDCs’ transmission power is only limited by their own transmission capability.

In definitions 2 and 3, the CgSCR and BgSCR are related to the operation point under which the HVDCs’ power is incompletely controllable and the MIDC system reaches its static voltage stability boundary (the saddle-node bifurcation). It is worth mentioning that, the controllable power at the boundary is a hyperplane whose normal vector is the right eigenvector of the minimum eigenvalue of \( J_{eq} \). This can be derived as follows.

At the boundary, the determinant of the matrix \( J_{MIDC} \) is zero,

\[ \text{det}(J_{MIDC}) = 0 \Rightarrow \]  

(29)

since,

\[ \text{det}(J_{eq}) = J_{eq}^{-1} \]  

(30)

Whether the HVDCs’ power is controllable is equivalent to the solution of algebraic equations,

\[ \Delta P_d = J_{eq} \Delta I_d \]  

(31)

Here, \( \Delta P_d \) is the desired power change and \( \Delta I_d \) is the corresponding current change. Linear algebra and matrix theory indicates that, the condition of solution to (31) is that the rank of the augmented matrix \( [J_{eq} \Delta P_d] \) is n-1, i.e., equals to the rank of matrix \( J_{eq} \). Namely, there exists a nonzero vector \([x_1, \ldots, x_n]\) which satisfies (32),

\[ x_{\text{eq}} \cdot \begin{bmatrix} J_{eq} & \Delta P_d \end{bmatrix} = 0 \]  

(32)

i.e.,

\[ \sum_{i=1}^{n} x_i \Delta P_i = 0 \]  

(33)

In (33), \([x_1, \ldots, x_n]\) is the left eigenvector to the zero eigenvalue of \( J_{eq} \), and it is also the right eigenvector to the minimum eigenvalue of \( J_{eq} \), whose elements are of same sign. The detailed proof is in Appendices E and F. Furthermore, (33) indicates that, the controllable power at the boundary is a hyperplane whose normal vector is \([x_1, \ldots, x_n]\). It is worth mentioning that, this normal vector (the right eigenvector to the minimum eigenvalue of \( J_{eq} \)) also indicate the critical direction of power change, which is useful to identify the nearest instability point and the stability boundary.

IV. COMPARISON OF gSCR WITH OTHER INDICES

In this section, we compare our defined gSCR with other indices recently proposed in the literature to assess grid strength of the MIDC systems, including the multi-infeed short circuit ratio (MSCR) in [9], the multi-infeed interactive short circuit ratio (MISR) in [8] and the equivalent effective short circuit ratio (EESCR) in [12].

A. Definition of MSCR and MISCR

In a MIDC system with \( n \) HVDCs, for the \( i \)-th HVDC inverter, MSCR, and MISCR, can be defined as follows,

\[ \text{MSCR}_i = \frac{1}{\sum_{j=1, j \neq i}^{n} P_j z_{ij}^{eq}} \]  

(34)

\[ \text{MISR}_i = \frac{U_j^{eq} z_{ij}}{P_j + \sum_{j=1, j \neq i}^{n} P_j} \]  

(35)
where \( P_i \) and \( U_i \) is the rated power and voltage of the \( i \)th HVDC and \( i \)th inverter-end ac bus; \( z_{ji} \) and \( z_{ij} \) are self-impedance and mutual-impedance of the \( i \)th converter bus respectively; \( S_{ac} = U_i^2 / z_{ii} \) is the short circuit capacity at the bus connected to the \( i \)th HVDC converter; \( MIIF_{ji} \) is the voltage interaction factor, which is defined as the ratio of the voltage percentage change at the bus connected with the \( j \)th HVDC converter to the one at the bus connected with the \( i \)th HVDC converter, i.e.,

\[
MIIF_{ji} = \left( \frac{\Delta U_i / U_i}{\Delta U_j / U_j} \right) = \frac{1}{z_{ji}} \left( \frac{U_j / U_i}{z_{ij} / z_{ji}} \right) \tag{36}
\]

It can be seen from (34) and (35), for a MIDC system with \( n \) HVDCs, it has \( n \) MSCRs and MISCRs.

**B. Relationship among MSCR, MISCR and gSCR**

Fig. 4 illustrates the relationship among various indices, including MSCR, MISCR and the proposed gSCR. As shown in Fig. 4, the MSCR and MISCR are derived from empirical or heuristic assumptions [8, 9]. However, gSCR are derived based on theoretical justification. And, both gSCR and MISCR can be reduced to MSCR under certain assumptions.

![Fig. 4 Relationship among MSCR, MISCR and gSCR](image)

At the rated operation point, \( U_i / U_i = 1 \), and MISCR in (35) is reduced to MSCR in (34) [12]. Similarly, gSCR can also be reduced to MSCR when meeting the condition of (38). Specially, the right eigenvector to the minimum eigenvalue of matrix \( J_{eq} \) satisfies,

\[
[p \Gamma^T - \mathbf{I} + \rho \mathbf{J}_{eq}^T] [x_1 \ldots x_n]^T = 0 \tag{37}
\]

If we add an additional assumption that

\[
x_i = x_2 = \ldots = x_n
\]

Then,

\[
\mathbf{J}_{eq} [x_1 \ldots x_n]^T = (\sum_{j=1,2,\ldots,m} P_{2j})^{-1} [x_1 \ldots x_n]^T
\]

By substituting (39) into (37), we have,

\[
MSCR = p \Gamma^T / 2 + \sqrt{\rho^2 \Gamma^2 + 4 + \rho^2} \tag{40}
\]

Comparing (40) with (27), we can see that \( MSCR \) is equivalent to gSCR under the additional assumption in (38). As shown in (40), MSCR is indeed related to static voltage stability. Thus, it is reasonable to assess grid strength of MIDC systems by using MSCR in [9]. However, if the assumption \( x_1 = x_2 = \ldots = x_n \) cannot be satisfied, the MSCR may not accurately assess grid strength of the MIDC systems. Based on this analysis, we define the symmetrical and unsymmetrical MIDC systems as follows:

**Definition4:** the symmetrical MIDC system is defined as a MIDC system in which the elements of right eigenvector to the minimum eigenvalue of \( J_{eq} \) are all same; otherwise, it is the unsymmetrical.

**Theorem1:** In a symmetrical MIDC system, for any HVDC \( i \), \( MSCR_i = gSCR_i \), and gSCR is not less than the minimum MSCR in general cases. The proof of Theorem1 can be found in Appendix G.

Based on Theorem1, it is worth mentioning that a real power grid is not generally a symmetrical system, so the static voltage stability analysis based on the minimum MSCR (denoted by \( MSCR_{min} \)) is always conservative (so as the \( MSCR_{min} \), since \( MSCR = MSCR_{min} \) where \( U_i / U_i = 1 \) is satisfied), which will be further verified in next section.

**C. Comparison of gSCR with MSCR and MISCR**

The proposed gSCR shares several features with MSCR and MISCR. Specifically, all these indices can assess grid strength of the MIDC systems and provide the information on the critical location. The MSCR and MISCR identify the critical location based on \( MSCR_{min} \) and \( MISCR_{min} \), while the gSCR identifies the critical location based on the maximum of the generalized participation factors (\( gPF_{max} \)) of the gSCR.

In addition to the common features above-mentioned, gSCR has the following advantages over the other indices:

a) The gSCR has a rigorous definition based on voltage stability analysis. Similar to MSCR and MISCR, the gSCR can consider the interactions among multiple HVDC inverters. But the gSCR is derived by the theoretical analysis of the relationship between SCR and voltage stability in SIDC systems and extending the analysis results to MIDC systems. However, the MSCR and MISCR are derived from empirical or heuristic assumptions.

b) The gSCR is the generalization presentation of SCR in MIDC systems and it has the same physical meaning of SCR. Thus, the critical gSCR (\( CgSCR \)) and boundary gSCR (\( BgSCR \)) can be used to distinguish very weak MIDC systems from weak ones and to distinguish strong MIDC systems from weak ones, respectively. Moreover, the \( CgSCR \) and \( BgSCR \) have the same values of \( CSCR \) and \( BSCR \), so the expected results shown in Fig. 2 are arrived. However, these results cannot be obtained by MSCR and MISCR.

c) The gSCR is derived based on modal analysis [13]. Based on gSCR, the information of the critical direction of power change (the right eigenvector of the gSCR of \( J_{eq} \)) can be provided to identify the nearest instability point and the stability boundary. However, MSCR and MISCR may not provide such information.

The comparisons of the gSCR with MSCR and MISCR are summarized and given in Table I.

<table>
<thead>
<tr>
<th>TABLE I COMPARISON OF gSCR WITH MSCR AND MISCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indices</td>
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<tr>
<td>Derivation</td>
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<tr>
<td>Has same value as SCR</td>
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<tr>
<td>Critical location</td>
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<tr>
<td>Assessment results</td>
</tr>
</tbody>
</table>

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D. Comparison of gSCR with EESCR

The equivalent effective short circuit ratio (EESCR) in [12] was defined as,
\[
EESCR = \frac{U_i^2/z_{ij}}{P_i + \sum_{j=1, n} NVIF_{ji} \cdot K_{ij} P_j} \tag{41}
\]
NVIF_{ji} is the nodal voltage interaction factor defined as,
\[
NVIF_{ji} = \left( \frac{U_i}{U_j} \right) \left( \frac{z_{ji}}{z_{ij}} \right) \tag{42}
\]

\[K_{ij} = \cos(\phi_{ij})/\cos(\phi_i)\] where \(\phi_{ij} = \text{atan}(P/Q)\); \(P_i\) and \(Q_i\) are the active and reactive power of the \(i^{th}\) HVDC, respectively; \(\delta_{ij}\) is the voltage angle between the \(i^{th}\) and \(j^{th}\) HVDC inverter-end ac bus; \(U_i\) is the voltage of the \(i^{th}\) inverter-end ac bus; \(z_{ii}\) and \(z_{ij}\) are self-impedance and mutual-impedance of the \(i^{th}\) converter bus considering the reactive power compensation capacitors, respectively.

The EESCR was proposed for grid strength assessment while taking into account the impact of the interactions of HVDC inverters. Also, the EESCR is derived from theoretical consideration. However, there are three main differences between the gSCR and EESCR: 1) EESCR is the extension of the concept of effective short circuit ratio from the SIDC systems to MIDC systems, while the gSCR is the extension of the concept of SCR; 2) The critical EESCR (CEESCR) may be variable for different power systems, while the CgSCR is exact and constant; 3) Only the critical location can be identified by EESCR, while both the critical location and critical direction of power change can be provided by gSCR. These differences will be further verified by simulation results in the next section.

V. SIMULATION VALIDATION

In this section, we validate the effectiveness of the proposed gSCR along with the CgSCR and BgSCR by using Matlab and DlgSILENT platforms to conduct numerical simulations in various MIDC systems, which are modified based on the HVDC benchmark model proposed by CIGRÈ in 1991. In the benchmark model, the RLC filters are replaced by compensation capacitors, and the parameters of the model is given in TABLE I.

<table>
<thead>
<tr>
<th>AC on rectifier side</th>
<th>AC on inverter side</th>
<th>DC system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>230kV</td>
<td>500kV</td>
</tr>
<tr>
<td>Power</td>
<td>1000MW</td>
<td>1000MW</td>
</tr>
<tr>
<td>Z</td>
<td>53.43Ω</td>
<td>250Ω</td>
</tr>
</tbody>
</table>

A. Comparison of gSCR with MSCR and MISCR

We first compare the gSCR with the MSCR and MISCR on a triple-infeed LCC-HVDC system, which can be represented with the model shown in Fig. 3. In this system, three SIDCs are interconnected by coupling impedances, at the rated operation point. The Thevenin equivalent reactance is chosen as \(z_{ij} = z_{ji} = 1/2.06\)p.u. and \(z_{ii} = z_{ji} = z_{ij} = 1.0\)p.u.; the rated transmission power of 1\(^{st}\) HVDC (i.e., \(P_{d1}\)) varies from \(0.01\)p.u. to \(1.31\)p.u.; the rated transmission power of 2\(^{nd}\) HVDC (i.e., \(P_{d2}\)) is calculated by ensuring the matrix \(\lambda_{max}\) Singular, same as the method proposed in [4], where this MIDC system operates at the static voltage stability boundary. Meanwhile, the rated transmission power of 3\(^{rd}\) HVDC (i.e., \(P_{d3}\)) maintains at \(1.0\)p.u.

In the system under the rated operation point (i.e., \(U_i/U_j = 1\)), MISCR is reduced to MSCR. However, gSCR cannot be reduced to MSCR according to Theorem 1 since the system is not symmetrical. Thus, the comparison of gSCR with MSCR and MISCR becomes the comparison of gSCR with MSCR.

The results are presented in Fig. 5. In Fig. 5(a), the trajectories of gSCR and MSCR at three inverter ac buses are changing with the power \(P_{d1}\). On these trajectories, each point is at the static voltage stability boundary. In Fig. 5(b), the trajectories of the gPFs related to three inverter ac buses are changing with the power \(P_{d1}\).

![Fig. 5 Trajectories of gSCR, MSCR and the gPFs with power \(P_{d1}\)](image)

It can be seen from Fig.5 that both MSCR and gSCR can identify critical locations in the system. As mentioned in TABLE I, the minimum of MISCR at each inverter ac bus (i.e., \(MSCR_{min}\)) is used to identify the critical location. As shown in Fig. 5(a), when \(P_{d1}<1.0\) p.u., the inverter ac bus 2 is the critical location among these ac buses since the inverter ac bus 2 has the minimum MISCR among three inverter ac buses. When \(P_{d1}>1.0\) p.u., the most critical becomes the inverter ac bus 1 since the inverter ac bus 1 has the minimum MISCR among three inverter ac buses. The same identification results are provided by gSCR. gSCR identifies the critical location by gPFs. Greater gPF means that an inverter ac bus is more critical. As shown in Fig. 5(b), when \(P_{d1}<1.0\) p.u., the inverter ac bus 2 is also the critical among these ac buses since it has the maximum gPF among three inverter ac buses. When \(P_{d1}>1.0\) p.u., the most critical also becomes the inverter ac bus 1 since the inverter ac bus 1 has the maximum gPF among three inverter ac buses.

Though both MSCR and gSCR identify the same critical locations in the system, the MSCR is conservative. In other words, gSCR is always not less than the \(MSCR_{min}\). As shown in Fig.
5(a), when $P_{di}<1.0$ p.u., the MSCR at inverter ac bus 2 (i.e., $MSCR_2$) is the minimum, and gSCR is larger than $MSCR_2$. When $P_{di}>1.0$ p.u., the MSCR at inverter ac bus 1 (i.e., $MSCR_1$) is the minimum, and gSCR is larger than $MSCR_1$. But when $P_{di}=1.0$ p.u., gSCR is equal to the MSCR at each inverter ac bus, which can be explained using Fig. 6.

Fig. 6 is a 3-D figure showing the elements of the right eigenvector to the minimum eigenvalue of $J_{a_d}$ at the static voltage stability boundary. In Fig. 6, the blue curve represents the trajectory of these elements when $P_{di}$ varies from 0.1p.u. to 1.31p.u.; the red and green curves represent the projections of this trajectory to XY and XZ surfaces, respectively.

![Fig. 6 Eigenvector of comprehensive Jacobi](image)

It can be seen from Fig. 6 that $MSCR$ at each inverter bus is equal to gSCR when all these elements of right eigenvector to the minimum eigenvalue of $J_{a_d}$ are same at the point A ($P_{di}$=1.0). All these elements of right eigenvector are equal to 0.577.

### B. Comparison of Critical and Boundary values of gSCR, MSCR, MISCR and EESCR

The comparisons focus on finding out whether the MIDC systems can operate stably at the rated operation point and the operation point where the COLAs of all HVDCs are 30°, by only the planning data. Here, the planning data means the rated transmission power of HVDCs, rated inverter-end ac voltage magnitude and Thevenin equivalent impedance. Thus, $U_i/U_d=1$ is satisfied. Under this condition, MSCR is reduced to MSCR; thus, MISCR and MSCR have the same critical and boundary values.

#### 1) Comparison of Critical Values

The critical value of gSCR (i.e., CgSCR) is compared with the critical values of MSCR, MISCR and EESCR on the triple-infeed LCC-HVDC system. This system is the same as the one used in Section V. A, but it has the following modifications: 1) the rated transmission power of all HVDCs is 1.0p.u.; 2) Thevenin equivalent reactance is chosen as $z_1=1/1.91$ p.u., $z_2=1/4.19$ p.u., $z_3=1/6.91$ p.u.; and 3) $z_1$ varies from 1/0.5p.u. to 2.0p.u., and $z_2$ is calculated by ensuring the matrix $J_{mxy}$ singular.

Following the definition of the critical value of SCR (i.e., CSCR) in a SIDC system, we expect that 1) in this triple-infeed LCC-HVDC system, the critical values of gSCR, MSCR, MISCR and EESCR are their values when the static voltage stability boundary is reached at the rated operation point; 2) like CSCR, these critical values are also constant and are close to their declared critical values (CgSCR, CMSCR, critical MISCR and CEESCR are declared to be closed to 2, 2, 2 and 1.5 respectively [8, 12]) when the system changes its parameters. Since, MISCR is reduced to MSCR, the critical values of MSCR and MISCR are equal, and we use CMSCR to represent their critical values. Here, we compare CgSCR, CMSCR and CEESCR. The results are presented in Fig. 7, which shows the changes of CgSCR, CMSCR and CEESCR with Thevenin equivalent susceptance $1/z_1$.

It can be seen from Fig. 7 that CgSCR meets our expectation. It is a constant and equal to 2.06, which is very close to 2, when increasing Thevenin equivalent susceptance $1/z_1$. However, CMSCR and CEESCR are changing as increasing this susceptance $1/z_1$, and CMSCR is much smaller than 2 while CEESCR is much smaller than 1.5. In other words, the critical values of MSCR, MISCR and EESCR are different from their expected values.

![Fig. 7 Trajectories of CgSCR, CMSCR and CEESCR with susceptance 1/z1](image)
C. Influences of the Used Assumptions on gSCR

In practical systems, the resistances of transmission lines may not be ignored and the phase angles between the inverter sides should be considered (i.e., assumption 2 and assumption 3 are not satisfied). The influences of the assumptions on gSCR for common MIDCs are analyzed in this subsection.

Fig. 9 shows the procedure of evaluating the influence of impedance angle \( \theta \) on CgSCR. The similar procedure can be applied to the impact analysis of other parameters on the CgSCR and BgSCR. The \( J_{\text{mzys}} \) in Fig. 9 is given in Appendix H.

Consider a dual-infeed LCC-HVDC system, in which two SIDsCs are inter-connected by coupling impedances. The Thevenin equivalent reactance is chosen as \( z_1 = 1/2.5 \text{p.u.}; z_{12} = 1.0 \text{p.u.} \) and \( z_2 = 1/4.5 \text{p.u.} \); the rated transmission power of these two HVDCs is 1.0 p.u.

Firstly, we study the influence of assumption 3 on gSCR. The simulations are conducted by changing the impedance \( z_2 \) and ensuring the matrix \( J_{\text{mzys}} \) singular at the rated operation point and at the operation point where COLAs of all HVDCs are 30\(^\circ\), respectively, with the voltage angle \( \delta_{12} \) between two inverter-end ac buses varies from -30\(^\circ\) to 30\(^\circ\). The results are given in Fig. 10(a) and Fig. 10(b).

It can be seen from Fig. 10 that, the CgSCR is around 2 and the BgSCR is around 3, when considering the influence of changes in voltage angle. And for the typical range of voltage angle change (-30\(^\circ\)~30\(^\circ\)), the deviations of CgSCR and BgSCR are both less than 3\%. It means that, the gSCR with CgSCR and BgSCR is widely applicable for common MIDC systems, even considering the voltage angles.

Secondly, we study the influence of assumption 2 on gSCR. The simulations are conducted by changing the impedance \( z_2 \) and ensuring the matrix \( J_{\text{mzys}} \) singular at the rated operation point, and at the operation point where COLAs of all HVDCs are 30\(^\circ\), respectively, with the impedance angle varies from 70\(^\circ\) to 90\(^\circ\). The results are given in Fig. 11(a) and Fig. 11(b).

It can be seen from Fig. 11 that, the CgSCR is still around 2 and the BgSCR is around 3, when considering the influence of changes in impedance angle. And for the typical range of impedance angle change (70\(^\circ\)~90\(^\circ\)), the deviations of CgSCR and BgSCR are both less than 4\%. Thus, the gSCR with CgSCR and BgSCR is widely applicable for common MIDC systems, even considering the impedance angle.

In summary, Fig. 10 and Fig. 11 show that the results of CSCR and BSCR as shown in Fig. 2 are still valid for CgSCR and BgSCR, even when the assumptions are relaxed.

VI. CONCLUSION

In this paper, the gSCR was proposed for assessing grid
strength of MIDC systems by the theoretically analysis of the relationship between static voltage stability and grid strength. It was shown that the gSCR can resolve the ambiguity when directly applying SCR for SIDC systems to MIDC systems. Moreover, the CgSCR and BgSCR were defined based on the gSCR to characterize the stability boundaries of MIDC systems. Namely, for a MIDC system, the AC system is strong (or very weak) when the gSCR is large (or less) than the BgSCR (or CgSCR). Our future work will focus on extending the gSCR to the operation generalized effective short circuit ratio when the detailed operational conditions are available.

APPENDIX

A. Elements in matrix \( J_{\text{SIDC}} \)

Based on (1), the elements in matrix \( J_{\text{SIDC}} \) can be calculated as follows,

\[
J_{\mu,T} = \frac{\partial P_d}{\partial I_d} = \frac{\partial (3\sqrt{2}\pi N K I_d \cos \gamma - 3\pi N X I_d^2)}{\partial I_d} = 3\sqrt{2}\pi N K U cos \gamma - 6\pi N X I_d = U_d - 3\pi N X I_d
\]

(43)

\[
J_{\rho,T} = U \frac{\partial P_d}{\partial U} = \frac{\partial (3\sqrt{2}\pi N K I_d \cos \gamma - 3\pi N X I_d^2)}{\partial U} = 3\sqrt{2}\pi N K U cos \gamma = (U_d + 3\pi N X I_d)I_d
\]

(44)

\[
J_{U,T} = -U \frac{\partial Q_d}{\partial U} = \frac{\partial (-P_d \tan \varphi + \omega B U^2)}{\partial U} = -J_{\mu,T} \tan \varphi - P_d \frac{\partial \tan \varphi}{\partial U} + 2\omega B U^2
\]

(45)

\[
J_{Q,U} = \frac{\partial Q_d}{\partial I_d} = \frac{\partial (-P_d \tan \varphi + \omega B U^2)}{\partial I_d} = \frac{\partial (-P_d \tan \varphi + \omega B U^2)}{\partial I_d} = \frac{P_d}{\cos \gamma - 1} \frac{X I_d}{\sqrt{2} K U^2} + 2\omega B U^2
\]

(46)

Since, it is assumed that the reactive power absorbed by HVDC is compensated locally by compensation capacitor, i.e., \((U^2 - U E \cos \delta)/Z = 0\).

\[
J_{\rho,T} = \frac{\partial (P_d - U E \sin \delta)/Z}{\partial \delta} = -U E \cos \delta/\partial \delta = U^2 Z^{-1}
\]

(47)

\[
J_{\rho,U} = \frac{\partial (P_d - U E \sin \delta)/Z}{\partial U} = J_{\rho,T} - P_d
\]

(48)

\[
J_{Q,U} = \frac{\partial (Q_d - U^2 - U E \cos \delta)/Z}{\partial \delta} = -U E \sin \delta/\partial \delta = U^2 Z^{-1}
\]

(49)

\[
J_{Q,U} = \frac{\partial (Q_d - U^2 - U E \cos \delta)/Z}{\partial U} = U \frac{\partial Q_d}{\partial U} - (U^2 - U E \cos \delta)/Z
\]

(50)

B. Proof of parameters \( T_i \) and \( \rho_i \) in (10) and (20) being constant and being equal at a given operation point

\( T_i \) and \( \rho_i \) are related to the operation point of the \( i \)-th HVDC. It can be proved that for any \( i \) in a MIDC system, \( T_i \) and \( \rho_i \) are constant and are equal at a given specific operation point, i.e., \( \rho_1 = \rho_2 = \cdots = \rho_n = \rho \) and \( T_1 = T_2 = \cdots = T_n = T \). The proof is detailed in the following three steps:

Step 1: The commutation overlap angle \( \mu_i \) of the \( i \)-th HVDC can be calculated as

\[
\mu_i = -\alpha_i + \cos^{-1}\left(\cos \alpha_i - \sqrt{2} X I_d / (K_U \sin(m(\pi)) \right)
\]

(51)

where \( \alpha_i = \pi - \beta_i \) is the firing angle of the \( i \)-th HVDC; \( \beta_i \) is the trigger angle of the \( i \)-th HVDC; \( X \) is commutation reactance of the \( i \)-th HVDC; \( I_d \) is DC current of \( i \)-th HVDC; \( K_U \) is the ratio of transformer of \( i \)-th HVDC; \( U \) is the ac voltage of the \( i \)-th HVDC inverter-end bus; and \( m = \pi \) in three-phase full bridge.

With our assumption 1, the extinction angles of all HVDCs are set to be constant and equal. The firing angles of all HVDCs are also same since their commutation overlap angles are same.

Thus, intermediate variables \( K(c) \) and \( c_i = X I_d / (\sqrt{2} K U) \) of each HVDCs are equal. That is, \( K(c) \) is equal for each HVDC; \( U \) is the ac voltage of the \( i \)-th HVDC inverter-end bus; and \( m = \pi \) in three-phase full bridge.

Based on Step 1, \( c_i \) and \( \alpha(c) \) for each HVDC are equal, so we have for each HVDC \( X P_i / U_i \) in (53) is equal.

Step 2: The DC voltage \( U_i \) on inverter side of the \( i \)-th HVDC can be calculated as

\[
U_i = 2.34 U \cos \beta_i + 3\pi X I_d
\]

(52)

substituting (52) into expression of \( c \) yields

\[
c_i = \frac{X I_d}{\sqrt{2} K U} = \frac{X I_d}{\sqrt{2} K U} \frac{X P_i}{U_i}
\]

(53)

where \( \alpha(c) = \sqrt{2} K \left(2.34 \cos \beta_i + 3\pi / c_i K U \right) \); \( P_i \) is the DC active power of the \( i \)-th HVDC.

Based on Step1, \( c_i \) and \( \alpha(c) \) for each HVDC are equal, so we have for each HVDC \( X P_i / U_i \) in (53) is equal.

Step 3: Based on Assumption 1, parameters of HVDCs based on their individual power capacity are all same, so \( X P_i \) and \( B_i P_{20} \) are equal for each HVDC respectively, i.e., for each HVDC \( \rho_i = P_{20} / P_{20} \) (or \( B_i P_{20} \)) is equal and \( \omega B_i U_i / P_i = \omega B_i U_i / P_{20} \) is equal as well. As the result, for each HVDC, \( T \) represented in (54) is equal.

\[
T_i = \frac{2c_i K(c_i)}{\left(1 - 1/m(\pi) \right) + 2\omega B_i U_i / P_{20}}
\]

(54)

C. Some properties on \( J_{eq} \)

\( J_{eq} \) is defined as the extended Jacobian matrix and \( J_{eq} = -DB \)

(55)

\( D = \text{diag}^{-1}(P_{20}) \). Some properties on \( J_{eq} \) are given as follows.

Lemma: The matrix \( J_{eq} \) can be diagonalizable and all eigenvalues of the matrix \( J_{eq} \) are positive; the minimum eigen-
value of $J_{eq}$ is simple [18-20], which means its geometric multiplicity and algebraic multiplicity is one. Furthermore, all elements of its eigenvector are of same sign.

The detailed proof of Lemma 1 is removed in Appendix B.

D. Proof of Lemma 1

In a MIDC system, $P_i > 0$, so $D$ is a positive definite matrix, and $D^{1/2}$ is a symmetric positive definite matrix too. Obviously, $B$ is a diagonally dominant matrix, so $B$ and its matrix congruence $D^{1/2}BD^{1/2}$ are all negative definite. Therefore, all eigenvalues of $D^{1/2}BD^{1/2}$ are negative.

Since $D^{1/2}J_{eq}D^{1/2} = J_{eq}$, $J_{eq}$ is similar to $-D^{1/2}BD^{1/2}$, thus, all eigenvalues of $J_{eq}$ are positive. Based on the physical meaning of circuit, the inverse matrix of $-B$ is the impedance matrix $Z$ which is a positive matrix, thus $J_{eq1} = -B^{-1}D^{-1} = ZD^{-1}$ is irreducible and strictly positive [19].

Depending on the nature of positive matrix, spectral radius of $J_{eq1}$ is its simple eigenvalue, whose geometric multiplicity and algebraic multiplicity is one, namely, the minimum eigenvalue of $J_{eq}$ is simple. Furthermore, all elements of its eigenvector are of same sign [19].

E. Proof of the relationship between the left and right eigenvectors of the minimum eigenvalue of matrix $J_{eq}$

Based on (22), the right eigenvector $x_k$ of the minimum eigenvalue of matrix $J_{eq}$ satisfies,

$$ (\rho T \cdot 1 + \rho^2 J_{eq} - J_{eq}) x_k = 0 $$

Substitute $J_{eq} = DB$ into (56) yields,

$$ (\rho T \cdot 1 - \rho^2 B^{-1}D^{-1} + DB) x_k = 0 $$

Left multiplying (57) by $D^{-1}$ yields,

$$ (\rho T \cdot ID^{-1} - \rho^2 D^{-1}B^{-1}D^{-1} + B) x_k = 0 $$

Transposing (58) yields,

$$ x_k^T (\rho T \cdot ID^{-1} - \rho^2 D^{-1}B^{-1}D^{-1} + B)^T = 0 $$

Since, $ID^{-1},D^{-1}B^{-1}D^{-1}$ and $B^{-1}$ are all symmetric matrices, (59) can be rewritten as,

$$ x_k^T D^{-1} (\rho T \cdot 1 - \rho^2 D^{-1}B^{-1}D^{-1} + DB) = 0 $$

Thus, $x_k^T = x_k^T D^{-1}$.

F. Proof of the critical power change direction

According to (18), the matrix $J_{MIDC}$ can be represented as,

$$ J_{MIDC} = J_c \left[ J_{eq} 0 \\ 0 J_{max} \right] J_b $$

where,

$$ J_c = \begin{bmatrix} I_n \\ 0 \\ J_{eq} \end{bmatrix}, \quad J_b = \begin{bmatrix} I_n \\ 0 \\ 0 \end{bmatrix} $$

When the static voltage instability occurs, the Jacobian matrix $J_{MIDC}$ becomes singular, i.e., there exist a zero eigenvalue of matrix $J_{MIDC}$, the left eigenvector of this zero eigenvalue can be represented as, $x_{eq} = [x_{La}, x_{Lb}, x_{Le}]$, which satisfies,

$$ x_{eq}^T J_{MIDC} = 0 $$

Substituting (21) and (61) into (62) yields,

$$ x_{La}^T J_{eq} + x_{Lb}^T J_{b,eq} + x_{Le}^T J_{eq,0} = 0 $$

$$ x_{eq}^T D^{-1} (\rho T \cdot 1 + \rho^2 J_{eq} - J_{eq}) = 0 $$

$$ x_{eq}^T = -x_{eq}^T \frac{\rho}{\lambda_1} $$

It can be seen from (63),

$$ x_{eq}^T D^{-1} = x_{eq}^T x_{eq} D^{-1} $$

where, $x_{eq}^T$ and $x_{eq}^T$ are the left and right eigenvector of the minimum eigenvalue of matrix $J_{eq}$. It can be also seen that,

$$ x_{eq}^T = x_{eq}^T \frac{\rho}{\lambda_1} - x_{eq}^T J_{eq} J_{eq}^T $$

$J_{eq} J_{eq}^T$ is a diagonal matrix, and all its diagonal elements are equal. Thus, $x_{eq}^T$ represents the same direction as $x_{eq}^T$.

According to (29), the matrix $J_{MIDC}$ can also be represented as,

$$ J_{MIDC} = J_c \left[ J_{eq} 0 \\ 0 J_{ac} \right] J_D $$

where,

$$ J_c = \begin{bmatrix} I_n \\ 0 \\ J_{eq} \end{bmatrix}, \quad J_a = \begin{bmatrix} 1_n \\ 0 \end{bmatrix} $$

Substituting (66) into (62) yields,

$$ x_{eq}^T J_{eq} = 0 $$

Thus, $x_{eq}^T$ is the left eigenvector to the zero eigenvalue of $J_{eq}$, and it is also the right eigenvector to the minimum eigenvalue of $J_{eq}$, whose elements are of same sign.

G. Proof of Theorem 1

Sum of each row to $J_{eq}^T$ is equal to $(MSCR)_{-1}$, when all elements of right eigenvector to the minimum eigenvalue of $J_{eq}$ are same, $MSCR_1 = MSCR_2 = \ldots = MSCR_{m}$ is satisfied based on (37)-(40). Thus $MSCR_{1}$ is a stochastic matrix and its maximum eigenvalue is 1 [19], i.e.,

$$ MSCR = \frac{1}{n} \max (J_{eq}^T) = gSCR $$

Equation (51) shows that each MSCR is equal to gSCR in the symmetrical system.

Let $a_{ij}$ denote the element of $(J_{eq}^T)^{-1}$, and assume $MSCR_{ij} = \min_{1,2,\ldots,m} MSCR_{ij}$. All eigenvalues satisfy (69) by the Gershgorin disk theorem [19],

$$ |z - a_{ij}| \leq \sum_{j \neq k} |a_{ij}| $$

Since gSCR is equal to the reciprocal of the maximum eigenvalue to $(J_{eq}^T)^{-1}$, it follows from (70) that

$$ gSCR^{-1} = \frac{1}{n} \max (J_{eq}^T) \leq MSCR^{-1} $$

$$ gSCR \geq MSCR_{ij} = \min_{1,2,\ldots,m} MSCR_{ij} $$

H. Calculating of $J_{max}$ and gSCR in fluence analysis

(16) can be rewritten as [17],

$$ x_{La}^T J_{eq} + x_{Lb}^T J_{b,eq} + x_{Le}^T J_{eq,0} = 0 $$

$$ x_{eq}^T D^{-1} (\rho T \cdot 1 + \rho^2 J_{eq} - J_{eq}) = 0 $$

$$ x_{eq}^T = -x_{eq}^T \frac{\rho}{\lambda_1} $$
where $M_i$ represents $\sin \delta_i$; $N_j$ represents $\cos \delta_j$; $\delta_i$ is the voltage angle between $i^{th}$ and $j^{th}$ HVDC inverter-end ac bus; $U_i$ and $U_j$ are the ac bus voltage of the $i^{th}$ and $j^{th}$ inverter respectively; $G_{ij}$ and $B_{ij}$ are the real and imaginary parts of the Thevenin equivalent impedance $Z_{ij}$ respectively; $P_i$ is the active power output of the $i^{th}$ HVDC. $J_{PdUii}$ and $J_{QdUii}$ are the diagonal parts of the matrices $J_{PdU}$ and $J_{QdU}$ in (16). $J_{mys}$ can be calculated by substituting (73) into (19).

The gSCR is calculated by (28), and the susceptance $b_{ij}$ in matrix $B$ is $b_{ij} = \frac{1}{|Z_{ij}|}$.

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